The Equilibrium Coordination Problem in Elections with Costly Information *

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Abstract

Voters decide between two policies in a majority election. All voters share a commonly known preference type but are uncertain about which policy is better for them. They can acquire private information about the policy consequences at a cost. Thus, political information is a public good. In all equilibria, voters under-invest in political information. We identify another source of inefficiencies: equilibrium mis-coordination. When the marginal cost of information are arbitrarily low, there are two equilibria, ordered by the information effort of the voters. In a large electorate, only the high effort equilibrium is asymptotically efficient. The result makes a case for information initiatives not only targeted at cost reductions but also at stimulating a culture of high informational effort.

We revisit the question of information acquisition of voters; in particular, the classical model of Martinelli (2006). Voters (or citizens) decide between two policies, A and B, in a majority election. All voters share a commonly known preference type but are uncertain about which policy is better for them. They can acquire private information about the policy consequences at a private cost. Thus, political information is a public good.

The public good nature of political information implies that voters will under-invest into information. This under-investment can be severe when the electorate

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is large: Then, each individual voter has a negligible probability of affecting the outcome and may not undertake sufficient costly efforts to get informed, so that election outcomes are not consistent with the preferences of the voters under full information and, thus, welfare-inefficient. This observation of informational inefficiency goes back to Downs *et al.* (1957).

Martinelli (2006) has made the observation that, while a voter's individual incentives to acquire may be small in a large electorate, the election may nevertheless aggregate the small but many pieces of information across the large number of voters so that outcomes are full-information equivalent and efficient. His work elegantly formalizes the "horse-race" between a growing number of voters and the information acquired by each individual voter. He identifies the critical condition for the existence of sequences of equilibria, indexed by the number of voters, that are asymptotically efficient. In other words, the conditions under which the under-investment problem is not too "severe" as to impact the possibility of efficient outcomes in large elections.[1]

This paper shows that the public good nature of information in voting settings implies another source of inefficiencies, besides that of under-investment. There is an equilibrium coordination problem. Namely, given Martinelli (2006)'s condition, there exist asymptotically efficient equilibrium sequences but also (non-trivial) inefficient equilibrium sequences.[2] The coordination problem is severe in the sense that the inefficient equilibrium sequences exist even when the cost of information are arbitrarily low. Further, they exist for a generic set of parameters.

The exact condition for existence is that the voters are not indifferent between the two policies given their prior belief. Without loss, we assume that they prefer policy A.[3] This "bias" towards A translates into equilibrium behaviour: In any (non-trivial) equilibrium, voters mix between voting for A without additional

[1]Following Martinelli (2006), the literature has analyzed in more depth the conditions on voter preferences and information cost that allow for efficient outcomes in large elections (Martinelli, 2007; Triossi, 2013; Oliveros, 2013). More distantly related work has focused on design features, such as optimal decision rules and committee size (see, e.g. Persico, 2004; Koriyama and Szentes, 2009).

[2]With *non-trivial*, we mean that in these equilibria, all voters acquire *some* information. There exist other trivial equilibria in which all citizens vote for the same policy, and no voters acquires *any* information.

[3]The case of indifference is non-generic. However, this does not mean that it is unimportant. In fact, many classical economic models, like the quasi-linear setting in the standard mechanism design with transfers framework, consider non-generic but yet important scenarios.

costly information on the one hand, and costly acquiring an informative binary signal $s \in \{a, b\}$ about the state and voting according to the realized signals (A after a and B after b) on the other hand.

The equilibria differ along the level of the voters' informational investment: In the inefficient equilibrium, the share of citizens that vote without costly information is higher than in the efficient equilibrium. Likewise, for those that acquire costly information, the quality of the acquired information is lower as well.

The multiplicity is driven by complementarities in the information acquisition behaviour, as follows: In the inefficient equilibrium, the higher share of those voting for A without information biases the election towards A so that A wins with a comparably high expected margin of victory in both states. Importantly, the incentives to costly acquire information vary with the closeness of the election and are lower when the election is less close. This way, the relatively low information acquisition in the inefficient equilibrium sustains itself. In contrast, in the efficient equilibrium, the election is more close to being tied in expectation, sustaining a higher level of information acquisition.

The equilibrium multiplicity may seem reminiscent of results in the literature on costly voting: e.g., the low and high turnout equilibrium in the participation games of Palfrey and Rosenthal (1983). This stream of literature has provided arguments showing that only one of these equilibria is "robust": in particular, the high turnout equilibrium is eliminated by strategic uncertainty about preferences or cost (Palfrey and Rosenthal, 1985), or about the number of voters (Myerson, 1998). In contrast, in our setting, both equilibria are robust to various forms of strategic uncertainty, as results in a companion paper show (Heese, 2022). In the companion paper, we propose a similar model to formalize the informational competition between political interest groups. The paper is general enough to embed a version of this paper's common interest model, with strategic uncertainty about preferences, information cost, and prior beliefs of the voters. The same equilibrium multiplicity persists.[4]

Our result may bear relevance for the design of initiatives targeted at the issue of badly informed electorates. Meta-analyses find no evidence that standard initiatives—door-to-door canvassing and digital information dissemination—that

[4] In the terminology of the companion paper: if all voters share a common interest under full information, they form one "interest group".

only target the reduction of information cost, have any effect (Dunning *et al.*, 2019). Our result makes a case for initiatives that are *also* targeted at stimulating a culture of high informational effort.

The rest of the paper is structured as follows: Section 1 restates the model as in Martinelli (2006). Section 2 characterizes the best response and the equilibrium conditions. Section 3 presents the main result. Section 4.2 dicusses the under-investment problem, including an sketch of the "horse-race"-argument behind Martinelli (2006)'s efficiency result. Section 5 presents a sketch of proof for the main result. Section 6 has concluding remarks.

1 Model

The following restates the model from Martinelli (2006) in its original notation: There are $2n+1 \geq 3$ voters (or citizens), two policies, A and B, and a binary state $z \in \{z_A, z_B\}$. The voters hold a common prior. The prior probability of state z_A is $q_A(0,1)$. The prior probability of the state z_B is $q_B = 1 - q_A$. Voters receive a utility of $U(d,z)$ if the outcome is d and the state z. We denote $U(A,z_A) - U(B,z_A) = r_A$ and $U(B,z_B) - U(A,z_B) = r_B$ and assume that $r_A, r_B > 0$. So, all voters prefer A in z_A and B in z_B. Further, we assume $q_A r_A > q_B r_B$ so that the outcome z_A maximizes the expected utility of all voters given the prior belief.[5]

The timing is as follows: Each voter chooses the precision $x \in [0, \frac{1}{2}]$ of her binary, private signal $s \in \{s_A, s_B\}$, that is $\frac{1}{2} + x = \Pr(s_A|z_A) = \Pr(s_B|z_B)$. When choosing precision x, the voter bears a cost $C(x)$. We assume that C is three times continuously differentiable, $C(0) = 0$, and $C'(x), C''(x), C'''(x) > 0$ for $x > 0$. The state and private signals realize. After observing the private signals, all citizens vote simultaneously. Finally, the outcome is decided by simple majority rule.

A pure strategy is a triple (x, v_A, v_B) where $x \in [0, \frac{1}{2}]$ specifies the precision choice, v_A specifies which policy citizens vote for after observing signal s_A, and v_B specifies which policy citizens vote for after observing signal s_B. A mixed strategy α is a probability distribution over the set of pure strategies. A *voting equilibrium*

[5] The counter-case $q_A r_A < q_B r_B$ yields qualitatively the same results. Just in the knife-edge case $q_A r_A = q_B r_B$, the inefficient equilibrium sequence does not exist.

is a symmetric mixed Bayes Nash equilibrium. A voting equilibrium that has a strategy (x, v_A, v_B) with $x > 0$ in its support is called *voting equilibrium with information acquisition.*

2 Best Response and Equilibrium Conditions

We characterize the best response. Fix one of the voters and the strategy α of the others. Whenever the votes of the other citizens do not split into n votes for A and n votes for B, the strategy of the fixed voter does not affect the outcome. Thus, when comparing the expected utilities from two strategies, it suffices to consider the "pivotal" counter-event. Denoting by $q(z;\alpha)$ the likelihood that a citizen votes for A in state $z \in \{z_A, z_B\}$ given α, the likelihood of the pivotal event is

$$\Pr(\text{piv}|z;\alpha) = \binom{2n}{n}\Big[q(z;\alpha)(1-q(z;\alpha)\Big]^n \tag{1}$$

for $z \in \{z_A, z_B\}$. Applying Bayes' rule,

$$\Pr(z|\text{piv};\alpha) = \frac{\Pr(z)\Pr(\text{piv}|z;\alpha)}{\sum_{z\in\{z_A,z_B\}}\Pr(z)\Pr(\text{piv}|z;\alpha)}. \tag{2}$$

The next Lemma shows that the pivotal likelihoods (1) completely characterize the best response.

Lemma 1 *Let* $C'(0) = 0$. *For any* α *with* $q(z;\alpha) \in (0,1)$ *for* $z \in \{z_A, z_B\}$, *there is* $\underline{p}, \overline{p} \in [0,1]$ *and* $x : [\underline{p}, \overline{p}] \to [0, \frac{1}{2}]$ *so that the unique best response is*

- $(0, B, B)$ *if* $\Pr(z_A|\text{piv};\alpha) < \underline{p}$,
- $(x(p), A, B)$ *if* $\underline{p} < \Pr(z_A|\text{piv};\alpha) < \overline{p}$,
- $(0, A, A)$ *if* $\underline{p} < \Pr(z_A|\text{piv};\alpha)$.

Further, $\overline{p}, \underline{p}$ *only depend on* $\Pr(z_A|\text{piv};\alpha)$ *and* $\Pr(z_B|\text{piv};\alpha)$.

Clearly, when the voter knows that the state is z_A, voting A without costly acquiring an informative signal about the state is optimal. Conversely, when the voter knows that the state is z_B, voting B is optimal. The lemma states that, in contrast, for beliefs p in an intermediate interval, acquiring some information $x(p)$ and "voting with the signal", i.e. for A after a and for B after b, is optimal.

Formally, let us compare the expected utility of a voter when following the strategy $(0, B, B)$, (x, A, B), or $(0, A, A)$. When choosing (x, A, B) this is

$$\begin{aligned} & \Pr(\text{piv}|z_A;\alpha)q_A\Big[U(A,z_A)(\frac{1}{2}+x)+U(B,z_A)(\frac{1}{2}-x)\Big] \\ + \;& \Pr(\text{piv}|z_B;\alpha)q_B\Big[U(B,z_B)(\frac{1}{2}+x)+U(A,z_B)(\frac{1}{2}-x)\Big]-C(x) \end{aligned} \tag{3}$$

plus the constant utility from the non-pivotal event. The expected utility from $(0, Y, Y)$ for $Y \in \{A, B\}$ is

$$\Pr(\text{piv}|z_A;\alpha)q_AU(Y,z_A)+\Pr(\text{piv}|z_B;\alpha)q_BU(Y,z_B). \tag{4}$$

plus the constant utility from the non-pivotal event. Subtracting $\Pr(\text{piv}|z_A;\alpha)q_AU(B,z_A)+\Pr(\text{piv}|z_B;\alpha)q_BU(A,z_B)$ from both expressions and re-arranging yields the indifference conditions

$$\begin{aligned} & \Big[\Pr(\text{piv}|z_A;\alpha)q_Ar_A+\Pr(\text{piv}|z_B;\alpha)q_Br_B\Big](\frac{1}{2}+x)-C(x) \\ = \;& \Pr(\text{piv}|z_Y;\alpha)q_Yr_Y, \end{aligned} \tag{5}$$

Evaluation of these indifference conditions shows the interval structure of the best response as in Lemma 1. To see why, fix $\Pr(\text{piv};\alpha) = \Pr(\text{piv}|z_A;\alpha)q_A + \Pr(\text{piv}|z_B;\alpha)q_B$ and divide both sides of (5) by it. For $y = A$, the difference of the left and right hand side of (5) is decreasing in $\Pr(z_A|\text{piv};\alpha) = \frac{\Pr(z)\Pr(\text{piv}|z;\alpha)}{\Pr(\text{piv}|z_B;\alpha)}$. Thus, for any $x > 0$, there is unique $p \in (0, 1)$ so that the voter is indifferent between (x, A, B) and $(0, A, A)$. Similarly, there is unique $p \in (0, 1)$ so that the voter is indifferent between (x, A, B) and $(0, B, B)$. Further details of the proof of Lemma 1 are relegated to the Appendix.

Equilibrium Conditions. The bias towards A given the prior, $q_Ar_A > q_Br_B$ will imply that equilibrium behaviour is shifted towards voting A: In any voting equilibrium with information acquisition, voters mix between a strategy (x, A, B) and $(0, A, A)$: Such equilibria are equivalently characterized by pairs $(x, \delta) \in (0, \frac{1}{2}] \times (0, 1]$ so that the mixed strategy $\alpha(x, \delta)$, in which the agent chooses the pure strategy (x, A, B) with probability $(1 - \delta)$ and $(0, A, A)$ with probability δ, satisfies two conditions. The first equilibrium condition (6) states that the marginal cost of acquiring the information precision $x > 0$ must equal the marginal benefit of doing so:[6]

$$\Pr(\text{piv}|z_A;\alpha)q_A r_A + \Pr(\text{piv}|z_B;\alpha)q_B r_B \;\;=\;\; C'(x), \tag{6}$$

or, equivalently, that the derivative of the expected utility of (x, A, B) with respect to x is zero. The second equilibrium condition states that the voters are indifferent between the strategies (x, A, B) and $(0, A, A)$, that is, (5) holds for $y = A$ and $x = x(\overline{p})$. The proof of this equilibrium characterization can be found in Martinelli (2006).[7] In the following, we identify voting equilibria with information acquisition with pairs $(x, \delta) \in (0, \frac{1}{2}] \times (0, 1]$.

3 Main Result

An equilibrium sequence $(\alpha_n)_{n\in\mathbb{N}}$ has *asymptotically efficient outcomes* if both the likelihood that A gets elected in z_A and the likelihood that B gets elected in z_B converge to 1 as $n \to \infty$, given $(\alpha_n)_{n\in\mathbb{N}}$, and it has *asymptotically inefficient outcomes* otherwise.

Theorem 1 *Let $C'(0) = C''(0) = 0$.*

1. *There is an equilibrium sequence with asymptotically efficient outcomes.*
2. *There is a sequence of voting equilibria with information acquisition that has asymptotically inefficient outcomes*

The existence of an equilibrium with asymptotically efficient outcomes given $C'(0) = C''(0) = 0$ is shown in Martinelli (2006).[8]

[6]To obtain (6), take the first-order derivative of the left hand side of (6) and set it equal to zero.

[7]See the proof of Theorem 4 and Theorem 1 therein.

[8]Martinelli (2006) also shows that the aggregate cost converge to zero in the equilibrium with efficient outcomes as $n \to \infty$. This implies that even when taking into account the cost of information, outcomes are approximately utilitarian efficient in large electorates. Strictly speaking, the argument is only provided for the case when $q_A r_A = q_B r_B$. However, the same argument generalizes to the case $q_A r_A > q_B r_B$.

The observation of an equilibrium multiplicity, with an inefficient equilibrium is novel. None of the previous literature has suggested that such an equilibrium would exist.

The proof of the novel result of Theorem 1 is in Appendix 7.3. It shows that, given the inefficient sequence of equilibria with information acquisition, the likelihood that the policy A is elected, converges to 1 as $n \to \infty$. Thus, the equilibrium sequence is inefficient since *all* voters strictly prefer policy B over A in state z_B.

The inefficient equilibrium is sustained by the following logic: There is a comparably high share of voters choosing the prior-optimal strategy $(0, A, A)$. Unlike the asymptotically efficient equilibrium, it satisfies $\delta > 2x$ when n is large. This condition implies that A receives a larger vote share in *both* states and that the election is more close to being tied in state z_B than in z_A, making $(0, A, A)$ a less attractive strategy; compare to Lemma 1.[9] In equilibrium, the share of voters choosing $(0, A, A)$ exactly offsets the prior preference for $(0, A, A)$ so that voters are indifferent between $(0, A, A)$ and (x, A, B).

Theorem 1 highlights that even if all voters share a common preference type and the marginal cost of information are arbitrarily low, voters may mis-coordinate on an inefficient equilibrium. It shows that the public good nature of political information creates to sources of inefficiencies: first, there is an inefficiency from "under-investment" into political information relative to the socially optimal participation behaviour in all equilibria. This is well-known from the literature (Martinelli, 2006). Second, there is an equilibrium coordination problem. This second problem persists even when the marginal cost of information are arbitrarily low. In contrast, given $C''(0) = 0$, when the electorate grows large, in the efficient equilibrium sequence, the individual investments of each voter become small sufficiently "slow" relative to the growing number of voters so that aggregating the pieces of information across all voters, the election outcomes are almost always optimal.

We sketch the intuition behind this "horse-race" argument in Section 4.2. In Section 5, we outline the logic of the existence argument for the inefficient equilibrium in sequence in detail.

[9]For any strategy α given by a pair $(x, \delta) \in (0, 1)^2$, the likelihood that a citizen votes A in z is given by $q(z_A; \alpha) = (1 - \delta)(\frac{1}{2} + x) + \delta$ and by $q(z_B; \alpha) = (1 - \delta)(\frac{1}{2} - x) + \delta$ respectively. A simple calculation shows that $\delta > 2x$ implies $\frac{1}{2} < q(z_B; \alpha) < q(z_A; \alpha)$.

4 The Under-Investment Problem

Political information is a public good in this setting. If a voter acquires information, she is bearing the cost privately, while all voters with the same interest benefit from her casting a more informed ballot.

As a consequence, in any equilibrium, the voters acquire less information than the welfare-maximizing symmetry strategy would prescribe. That is, there is under-investment.[10]

The basic intuition behind this under-investment problem goes back to Downs' argument that a rational voter should only require very little or no costly information in a large electorate, which can be formalized as follows: When the electorate grows large, $n \to \infty$, the likelihood that the vote of a single citizen affects the outcome, goes to zero. As a consequence, the voters' precision x—solving (6)—goes to zero as well. If, on the contrary, the voter would take into account the utilitarian welfare of *all* voters, this would *not* be an immediate implication. In this case, the voters' choice of precision would equate

$$(2n+1)\Big[\Pr(\mathrm{piv}|z_A;\alpha)q_A r_A + \Pr(\mathrm{piv}|z_B;\alpha)q_B r_B\Big] \quad = \quad C'(x), \tag{7}$$

given the strategy α of the others; compare to (6). Now, if α implies vote shares sufficiently close to $\frac{1}{2}$, i.e., $q(z;\alpha) \approx \frac{1}{2}$, the pivotal likelihood is approximately of the order $n^{-\frac{1}{2}}$.[11] and the socially optimal precision would *not* go to zero as $n \to \infty$.[12]

Importantly, as Martinelli (2006) has shown, given the condition $C''(0) = 0$ from Theorem 1, asymptotically efficient equilibria exist *despite* the under-investment problem. In this sense, under this condition, the under-investment is not severe. In Section 4.2, we provide a compact argument sketching the intuition behind this result and why $C''(0) = 0$ is the critical condition. For simplicity, we consider the polynomial cost functions $C'(x) = x^d$ so that $C''(0) = 0$ is equivalent to $d > 2$. The argument shows that, if $d > 2$, any sequence of equilibria α_n with $q(z_A;\alpha_n) \geq \frac{1}{2}$ and $q(z_B;\alpha_N) \geq \frac{1}{2}$ must have asymptotically efficient outcomes.

[10] We show this formally in the Appendix. For simplicity, we provide the argument only for the case when $q_A r_A = q_B r_B$ that is treated centrally in Martinelli (2006).

[11] Compare to the Stirling approximation of the pivotal likelihood in the Appendix, (28).

[12] A similar observation has been made by Evren (2012) who analyzed a model of costly voting with altruistic voters.

Section 4.1 prepares by introducing some notation.

4.1 Informativeness of Equilibrium Sequences

For any sequence of equilibria $(\alpha_n)_{n\in\mathbb{N}}$ given by a sequence of pairs $(x_n, \delta_n)_{n\in\ltimes}$ and any n, let

$$\delta_n(z;\alpha_n) = \frac{q(z;\alpha_n) - \frac{n}{2n+1}}{s(z;\alpha_n)}. \tag{8}$$

for $z \in \{z_A, z_B\}$. This measures the distance between the expected vote share and the majority threshold in multiples of the standard deviation $s(z;\alpha_n)$ of the vote share distribution for $z \in \{z_A, z_B\}$, where $s(z;\alpha_n)^{-1} = \sqrt{\frac{(2n+1)}{q(z;\alpha_n)(1-q(z;\alpha_n))}}$.[13] A normal approximation of the distribution of the number of A-votes shows that, as $n \to \infty$, the probability that A gets elected in ω converges to[14]

$$\lim_{n\to\infty} \Pr(A|z;\alpha_n) = \lim_{n\to\infty} 1 - \Phi(-\delta_n(z;\alpha)). \tag{9}$$

Here, $\Phi(\cdot)$ is the cumulative distribution of the standard normal distribution. So, the asymptotic distribution of the outcome policy only depends on $\lim_{n\to\infty} \delta_n(z;\alpha_n) \in \mathbb{R} \cup \{\infty, -\infty\}$.

We call $\lim_{n\to\infty} \delta_n(z_A;\alpha_n) - \delta_n(z_B;\alpha_n)$ the *informativeness* of an equilibrium sequence. The informativeness is positive if the aggregate effect of the voters' information acquisition on vote shares is large enough so as to impact outcomes. Precisely, given (9), this is a necessary condition for the asymptotic outcome distribution to be different in the two states.

[13] Let $q_n = q(z;\alpha_n)$. The number v_n of A-votes follows a Binomial distribution with variance $(2n+1)q_n(1-q_n)$. So, the vote share $\frac{v_n}{2n+1}$ of A follows a distribution with standard deviation $s(\omega;\sigma_n)$.

[14] Let $q_n = q(z;\alpha_n)$. Take the normal approximation $\mathcal{B}(2n+1, q_n) \simeq \mathcal{N}((2n+1)q_n, (2n+1)q_n(1-q_n))$ of the distribution of the number of A-votes. It shows that the probability that there are more A-votes than B-votes converges to $\lim_{n\to\infty} 1 - \Phi(\frac{(2n+1)(\frac{n}{2n+1} - q_n)}{((2n+1)q_n(1-q_n))^{\frac{1}{2}}}) = \lim_{n\to\infty} 1 - \Phi(-\delta_n(z;\alpha_n))$. Note that we are applying the Lindeberg-Feller version of the central limit theorem for the normal approximation, which also applies to triangular arrays of random variables.

4.2 The Possibility of Asymptotically Efficient Outcomes

Let $C''(0) = 0$. Take a sequence of equilibria $(\alpha_n) = (x_n, \delta_n)_{n\in\mathbb{N}}$ with $q(z_A; \alpha_n) \geq \frac{1}{2}$ and $q(z_B; \alpha_n) \leq \frac{1}{2}$. Suppose that the outcomes are not asymptotically efficient. Given (9), this implies that $\lim_{n\to\infty} \delta_n(z; \alpha_n)$ is finite for some state z. We proceed in two steps to establish a contradiction: First, we show that the informativeness of the equilibrium sequence is finite. Second, we establish, by the "horse-race" argument, that the informativeness of the equilibrium must be infinite.

Suppose that the informativeness is infinite. Thus, $\lim_{n\to\infty} \delta_n(z; \alpha_n)$ is finite in one state, e.g., $z = z_A$, but not in the other, $z = z_B$. We observe that the normal approximation (9) also holds locally,[15]

$$\lim_{n\to\infty} \Pr(\text{piv}|z;\alpha_n)(2n+1)s(z;\alpha_n) \quad = \quad \lim_{n\to\infty} \phi(\delta_n(z;\alpha_n)), \tag{10}$$

where ϕ the density of the standard normal distribution. Thus,

$$\lim_{n\to\infty} \frac{\Pr(z_A|\text{piv};\alpha_n)}{\Pr(z_b|\text{piv};\alpha_n)} = \lim_{n\to\infty} \frac{\Pr(z_A)}{\Pr(z_B)} \frac{\phi(\delta_n(z_A;\alpha_n))}{\phi(\delta_n(z_B;\alpha_n))} \tag{11}$$

diverges and voters become almost certain that the state is z_A conditional on being pivotal. But this implies that the best response is for all voters to vote A; compare to Lemma 1, which, in turn, implies $\lim_{n\to\infty} \delta_n(z; \alpha_n) = \infty$, contradicting the initial assumption. We conclude that the informativeness is finite.

Suppose that the informativeness is finite. What matters for the "informativeness" of the aggregate voting behavior is the distance between the expected vote share in the two states in terms of standard deviations,

$$\begin{aligned} \lim_{n\to\infty} \delta_n(z_A;\alpha_n) - \delta_n(z_B;\alpha_n) \quad &= \quad \lim_{n\to\infty} \frac{q(z_A;\alpha_n) - q(z_B;\alpha_n)}{s(z_A;\alpha_n)} \\ &= \quad \lim_{n\to\infty} \frac{2x(1-\delta)}{s(\alpha;\sigma_n)}. \end{aligned} \tag{12}$$

Here, we used the definition (8) of $s(z_A; \alpha_n)$ and that $\lim_{n\to\infty} \frac{s(z_A;\alpha_n)}{s(z_A;\alpha_n)} = 1$, which is implied by a finite informativeness. Hence, the relevant comparison is how

[15] The local central limit theorem is due to Gnedenko (1948). The version that we apply is the one for triangular arrays of integer-valued variables as in Davis and McDonald (1995), Theorem 1.2. Compare also to the equation (11) therein.

fast the precision x decreases relative to how fast the standard deviation of the vote share increases. The following shows that the critical condition in this "horse-race" is if $d > 2$.

Observe that, given $C(X) = \frac{x^d}{d}$ with $d > 2$, the precision acquired by the voters is of an order *larger* than the pivotal likelihood.[16]

This is a direct consequence of the first-order condition (6). Now, denote $s_n = s(z_A; \alpha_n)$ and $q_n = q(z_A; \alpha_n)$. Given (10), the pivotal likelihood is asymptotically proportional to $((2n+1)s_n)^{-1}$. Since $((2n+1)s_n)^{-1} = s_n(q_n(1-q_n))^{-1}$, it is asymptotically proportional to the standard deviation s_n.[17] Combining this with the first observation, we see that the standard deviation vanishes relative to the precision x_n of the voters. Finally, (12) implies that, under the best response, the informativeness diverges to infinity.

5 The Equilibrium Coordination Problem

In this section, we sketch the constrution of the inefficient equilibrium sequence of Theorem 1. The proof leverages a generalized version of the Poincaré-Miranda-Theorem (Miranda, 1940), a fixed point theorem equivalent to Brouwer's. The generalized version relaxes the condition under which the theorem applies. This relaxation is crucial for our purposes since the standard conditions are *not* satisfied.

Lemma 2 *(Generalized Poincaré -Miranda Theorem)*
Take any continuous $f, g : [0,1] \times [0,1] \to [-1,1]^2$. If

$$f(0,t) < 0 \quad \textit{for all } t, \tag{13}$$

$$f(1,t) > 0 \quad \textit{for all } t. \tag{14}$$

[16]The first-order condition (6) implies $x_n^{d-1} = \Pr(\text{piv}|z_A;\alpha)q_A r_A + \Pr(\text{piv}|z_B;\alpha)q_B r_B$. Thus, $\lim_{n\to\infty} \Pr(\text{piv}|z;\alpha) = \lim_{n\to\infty} x_n^{d-2} = 0$ for $d > 2$ since $x_n \to 0$ as $n \to \infty$, given that (6) and that the pivotal likelihoods converge to zero. The latter can be seen formally from the Stirling approximation in (28).

[17]Recall that $((2n+1)s_n)^{-1}$ is the standard deviation of the Binomial distribution of the number of vote shares. Note that $((2n+1)s_n)^{-1} = \Big[(2n+1)(q_n(1-q_n))\Big]^{-\frac{1}{2}} = s_n(q_n(1-q_n))^{-1}$ since $s_n = (\frac{(2n+1)}{q_n(1-q_n)})^{-\frac{1}{2}}$; see (8) and thereafter.

and

$$g(r,0) > 0 \quad \textit{if } f(r,0) = 0, \tag{15}$$

and

$$g(r,1) < 0 \quad \textit{if } f(r,1) = 0, \tag{16}$$

then, there is $(r^0, t^0) \in (0,1)^2$ *such that* $f(r^0, t^0) = g(r^0, t^0) = 0$.

The lemma clearly also holds on any compact domain $D \subseteq \mathbb{R}^2$ other than $[0,1]^2$.[18] The proof of Lemma 2 is provided in a companion paper with two co-authors, (Ekmekci *et al.*, 2022). The key step is to show that the first two conditions ensure that there is a continuous function $h : [0,1] \to [0,1]$ so that $f(h(t), t) = 0$ for all t. The third and fourth condition yield $g(h(0), 0) > 0$ and $g(h(1), 1) < 0$ and ensure that—by an application of the intermediate value theorem—there is t_0 so that $g(r^0, t^0) = 0$ for $h(t^0) = r^0$.

To construct the inefficient equilibrium sequence of Theorem 1, we apply Lemma 2 to the functions[19]

$$\begin{aligned} f(x,\delta) &= \max_{x \in [0,\frac{1}{2}]} \Pr(\text{piv}|z_B; \alpha(x,\delta)) q_B r_B (\frac{1}{2} + x) \\ &\quad - \Pr(\text{piv}|z_A; \alpha(x,\delta)) q_A r_A (\frac{1}{2} - x) - C(x), \qquad (17) \\ g(x,\delta) &= \text{MB}(x,\delta) - C'(x) \qquad (18) \end{aligned}$$

for $\text{MB}(x,\delta) = \Pr(\text{piv}|z_A; \alpha(x,\delta)) q_A r_A + \Pr(\text{piv}|z_B; \alpha(x,\delta))$ on an appropriate domain D_n. The domain $D_n = \{(x_n, \delta_n) : \delta_n \in \Delta_n, x_n \in X_n(\delta_n)\}$ is chosen so that for all $(x_n, \delta_n) \in D_n$, $1 > \delta_n > 2x_n > 0$. Thus, the implied vote shares

$$\begin{aligned} q(z_A; \alpha) &= (1-\delta)(\frac{1}{2} + x) + \delta, \\ q(z_B; \alpha) &= (1-\delta)(\frac{1}{2} - x) + \delta \end{aligned}$$

are ordered as

$$\frac{1}{2} < q(z_B; \alpha_n) < q(z_A; \alpha_n). \tag{19}$$

[18]This is because any compact domain $D \subseteq \mathbb{R}^2$ is homeomorph to $[0,1]^2$.

[19]Note than f is continuous by an application of Berge's theorem of the maximum.

We show that the boundary conditions (13) - (16) are satisfied by f and g on D_n when n is large enough. Recalling the equilibrium conditions (5) and (6), the sequence of pairs $(x_n^0, \delta_n^0)_{n\in\mathbb{N}}$, given by Lemma 2, yields a sequence of voting equilibria with information acquisition. Given (19), and application of the law of large numbers implies that policy A is elected with probability larger than $\frac{1}{2}$ in both states as $n \to \infty$. Thus, the equilibrium sequence has asymptotically inefficient outcomes. The details are in Appendix 7.3. Below, we sketch the logic why the boundary conditions are satisfied. This will be instructive of the economic forces that hold together the inefficient equilibrium.

Sketch of the Logic. Observe that (12) shows that the precision x_n of the voters scales the distance between the vote shares in the two states. Given (10) and the ordering (19), when conditioning on the pivotal event, the voter's belief about the likelihood of the state being z_A is smaller when x_n is larger and larger when x_n is smaller.

We chose $X_n = \left[x_n^{\min}, x_n^{\max}(\delta_n)\right]$ so that for $x_n = x_n^{\max}(\delta_n)$ the implied belief about the likelihood of α is in $\left[\underline{p}, \overline{p}\right]$; compare to Lemma 1. Thus, the information acquisition strategy $(x(p), A, B)$ is preferred over $(0, A, A)$; or, in other words, $f(x_n^{\max}(\delta_n), \delta_n) > 0$. Likewise, for $x_n = x_n^{\min}$, the implied belief is sufficiently high so that $(0, A, A)$ is preferred over any information acquisition strategy (x, A, B); in other words, $f(x_n^{\min}, \delta_n) < 0$. In fact, we show that one can let $x_n^{\min} = 0$.

We chose $\Delta_n = \left[\delta_n^{\min}, \delta_n^{\max}\right]$ so that the following holds: For $\delta_n = \delta_n^{\max}$, the vote shares are shifted towards A sufficiently much so that the pivotal likelihood and therefore the marginal benefit $\mathrm{MB}(x_n, \delta_n)$ from acquiring information are exponentially small. This way, $g(x_n, \delta_n) = 0$ implies $x_n \approx 0 = x_n^{\min}$ when n is large enough. Given that $f(x_{\min}, \delta_n) < 0$, any x_n' with $f(x_n', \delta_n) = 0$ is larger than $x_{\min}$ so that the convexity of the cost function C implies $g(x_n', \delta_n) < 0$. For $\delta_n = \delta_n^{\min}$, there is much less bias towards A and the implied vote shares are sufficiently close to $\frac{1}{2}$ in at least one state. We choose $\delta_n^{\min}$ so that the marginal benefit $\mathrm{MB}(x_n, \delta_n)$ from acquiring information is so large that $g(x_n', \delta_n) = 0$ implies $x_n' > x_n^{\max}$. Since C is convex, $g(x_n, \delta_n) < 0$ for any $x_n \leq x_n^{\max}$; in particular, for any $x_n \leq x_n^{\max}$ which additionally satisfies $f(x_n, \delta_n) = 0$.

Lemma 10 thus yields an equilibrium (x_0, δ_0) when n is large enough, and, following the logic sketched above, the information investment x_0 in this equilibrium is so that the voter is indifferent between choosing $(0, A, A)$ (i.e. voting according to the prior belief) and choosing (x_0, A, B). The likelihood of voting consistent with the prior belief, δ_0 is so that the incentives to acquire information are just so that x_0 is the optimal precision acquired by a voter under the best response.

6 Conclusion

We have revisited the question of information acquisition in elections; and, in particular, the classic model of Martinelli (2006).

The main insight of our analysis is that when political information is a public good, complementarities arise that create an equilibrium coordination problem. Equilibria differ in the extent and quality of informational efforts of the voters: There is an equilibrium with relatively higher and another with lower efforts. Importantly, we have shown that the equilibrium outcomes of the low effort equilibrium are inefficient even when information cost are arbitrarily low and there are arbitrarily many voters that acquire information. None of the previous literature has suggested that such an equilibrium would exist.

Our result highlights that it might not be easily possible to address the problem of badly informed electorates simply through informational initiatives and campaigns targeted at reductions of information cost. Indeed, meta-analyses find no evidence that standard cost reduction initiatives—door-to-door canvassing and digital information dissemination—have any effect (Dunning *et al.*, 2019). Going forward, the result may be understood as qualifying initiatives that *also* aim at stimulating a culture of high levels of informational effort.

7 Appendix

7.1 Proof of Lemma 1

Given any α with $q(z; \alpha) \in (0, 1)$ for $z \in \{z_A, z_B\}$, the pivotal likelihood is non-zero. We claim that, given α, the expected utility from choosing (x, A, B) has a unique maximizer $x > 0$. To see this, take the derivative of the expected utility from (x, A, B), or equivalently of the left hand side of (5), to obtain

$$\Pr(\text{piv}|z_A;\alpha)q_A r_A + \Pr(\text{piv}|z_A;\alpha)q_B r_B - C'(x). \tag{20}$$

Clearly, there is a unique $x > 0$ that satisfies the first-order condition, since $C'(0) = 0$ and $C''(x) > 0$ for $x > 0$. Fix $\Pr(\text{piv}|z_A;\alpha) + \Pr(\text{piv}|z_A;\alpha)$ and denote by $x(p)$ the maximizer maximizer $x(p)$ as a function of $p = \Pr(z_A|\text{piv};\alpha)$ in the following. Note that the only possible strategies in the support of the best response to α are $(0, A, A)$, $(0, B, B)$, and $(x(p), A, B)$.

The voter prefers $(0, A, A)$ over $(0, B, B)$ whenever $\Pr(\text{piv}|z_A;\alpha(x,\delta))q_A r_A > \Pr(\text{piv}|z_B;\alpha(x,\delta))q_B r_B$ and vice versa. So, there is a unique belief $p^* \in (0, 1)$ at which the voter is indifferent between $(0, A, A)$ and $(0, B, B)$. At this belief, the voter is also indifferent between $(0, A, A)$ and $(0, A, B)$ given (5); hence, she strictly prefers $(x(p), A, B)$ over $(0, A, A)$ and $(0, B, B)$.

Now, the difference between the right hand and left hand side of (5) is increasing in $p = \Pr(\text{piv}|z_A;\alpha)$, for $\Pr(z_A|\text{piv};\alpha) < p^*$ and $y = B$, as already observed in the main text. Thus, there is a unique belief $\underline{p} < p^*$ at which the voter is indifferent between $(0, B, B)$ and $(x(p), A, B)$. For $p < \underline{p}$, $(0, B, B)$ is the unique best response. Similarly, for $\Pr(z_A|\text{piv};\alpha) > p^*$ and $y = A$, the difference between the right hand and left hand side of (5) is decreasing in $p = \Pr(\text{piv}|z_A;\alpha)$. Thus, there is a unique belief $\overline{p} > p^*$ at which the voter is indifferent between $(0, B, B)$ and $(x(p), A, B)$. For $p > \overline{p}$, $(0, A, A)$ is the unique best response. Finally, the above shows that for $p \in (\underline{p}, \overline{p})$, the strategy $(x(p), A, B)$ is the unique best response.

7.2 Formal Derivation of Under-Investment given $q_A r_A = q_B r_B$

Suppose that $q_A r_A = q_B r_B$. It is well-known that under this condition there is a unique equilibrium α^* in which all voters use a strategy (x^*, A, B). The precision x^* solves (5), that is,[20]

$$\Pr(\text{piv}|z_A;\alpha^*)q_A r_A + \Pr(\text{piv}|z_B;\alpha^*)q_B r_B \;=\; C'(x), \tag{21}$$

[20] See Theorem 1 in Martinelli (2006).

Now, suppose, in contrast to the assumption of the model in Section 1 that all voters internalize the information externalities and maximize social welfare. First, by much the same argument as in Martinelli (2006), there is a *unique* equilibrium α^{**} n which all voters use a strategy (x^{**}, A, B). The precision x^{**} solves

$$n\Big[\Pr(\text{piv}|z_A;\alpha^{**})q_A r_A + \Pr(\text{piv}|z_B;\alpha^{**})\Big]q_B r_B \quad = \quad C'(x^{**}), \tag{22}$$

It follows from a result by McLennan (1998) that the welfare-maximizing symmetric strategy is a symmetric equilibrium of the game of the voters. Consequently, the welfare-maximizing strategy is identical to α^{**}.

Suppose that voters do not under-invest into information relative to the welfare-maxiziming strategy, that is, $x^* \geq x^{**}$. Then,

$$\text{MB}(\alpha^*) \leq \text{MB}(\alpha^{**}). \tag{23}$$

for $\text{MB}(\alpha) = \Pr(\text{piv}|z_A;\alpha)q_A r_A + \Pr(\text{piv}|z_B;\alpha)q_B r_B$. It follows that $\text{MB}(\alpha^*) < n\text{MB}(\alpha^{**})$ and this implies $x^* < x^{**}$ since $C''(x) > 0$ for $x > 0$.

7.3 Proof of Theorem 1

In the following, we consider candidate equilibrium strategies α_n, given by pairs $(x_n, \delta_n) \in D_n$ for the domain

$$D_n = \{(x,\delta) : \delta \in \left[3M_n(\epsilon)n^{-\frac{1}{2}}, \epsilon\right], x \in \left[0, M_n(\delta)n^{-\frac{1}{2}}\right]\} \tag{24}$$

for some $\epsilon > 0$ and an increasing function $M_n : [0,\epsilon] \to \mathbb{R}_{>0}$ that will be defined in the course of the proof. Often, we only specify one of the variables of the pair, e.g., only x_n, to then establish a claim for all strategies corresponding to pairs $(x'_n, \delta_n) \in D_n$ with $x_n = x'_n$. Further, we use the notation x^*_n for the unique precision x that maximizes the expected utility across all strategies (x, A, B). That is, x^*_n solves (5).

Note that for any strategy α given by a pair (x, δ)

$$q(z_A; \alpha) = (1-\delta)(\frac{1}{2} + x) + \delta, \tag{25}$$

$$q(z_B; \alpha) = (1-\delta)(\frac{1}{2} - x) + \delta \tag{26}$$

Note that D_n is homeomorph to $[0,1]^2$ by a homeomorphism h that maps the boundaries as follows:

$$\begin{aligned}
\{0\} \times \left[3M_n(\epsilon)n^{-\frac{1}{2}}, \epsilon\right] &\to \{0\} \times [0,1]\,, \\
\{M_n(\delta)n^{-\frac{1}{2}}\} \times \left[3M_n(\epsilon)n^{-\frac{1}{2}}, \epsilon\right] &\to \{1\} \times [0,1]\,, \\
\left[0, M_n(\delta)n^{-\frac{1}{2}}\right] \times \{3M_n(\epsilon)n^{-\frac{1}{2}}\} &\to [0,1] \times \{0\}, \\
\left[0, M_n(\delta)n^{-\frac{1}{2}}\right] \times \{\epsilon\} &\to [0,1] \times \{1\}.
\end{aligned}$$

In the following, we establish the conditions of the generalized Poincare-Miranda Theorem (Lemma 2) for the functions $f \circ h^{-1}$ and $g \circ h^{-1}$ with f, g as in (17), one-by one.

To show (13), we let $x_n = K_n n^{-\frac{1}{2}}$ for some sequence $(K_n)_{n \in \mathbb{N}}$ with $K_n \to 0$ as $n \to \infty$ (note that this includes the case $x_n = 0$). This implies $\lim_{n\to\infty} \delta_n(z_A, \alpha_n) - \delta_n(z_B, \alpha_n) = 0$. So, given (11), the voters do not learn anything when conditioning on the pivotal event, as $n \to \infty$,

$$\lim_{n\to\infty} \Pr(z_A|\text{piv}; \alpha_n) = \Pr(\alpha). \tag{27}$$

A Stirling approximation of the pivotal likelihood, [21]

$$\Pr(\text{piv}|z; \alpha_n) \approx 4^n (n\pi)^{-\frac{1}{2}} \Big[q(z; \alpha_n)(1 - q(z; \alpha_n))\Big]^n, \tag{28}$$

shows that the pivotal likelihood goes to zero as $n \to \infty$. Thus, $x_n^* \to 0$. Since all voters strictly prefer A given the prior belief, (27) implies that for n sufficiently large, given x_n^*, after any signal $s \in \{a, b\}$, the voter strictly prefers A over B.

[21] Stirling's formula yields $(2n)! \approx (2\pi)^{\frac{1}{2}} 2^{2n+\frac{1}{2}} n^{2n+\frac{1}{2}} e^{-2n}$ and $(n!)^2 \approx (2\pi) n^{2n+1} e^{-2n}$. Consequently, $\binom{2n}{n} \approx (2\pi)^{-\frac{1}{2}} 2^{2n+\frac{1}{2}} n^{-\frac{1}{2}} = 4^n (n\pi)^{-\frac{1}{2}}$. Plugging this expression for the binomial coefficient into (25) and (26) yields $\Pr(\text{piv}|\omega; n) \approx 4^n (n\pi)^{-\frac{1}{2}} (q(1-q))^n$ for $q = q(z; \alpha_n)$.

Thus, (x_n^*, A, A) yields strictly more utility than (x_n^*, A, B). Further, $(0, A, A)$ yields strictly more utility than (x_n^*, A, A) since the voter does not costly acquire information. We conclude that all voters strictly prefer $(0, A, A)$ to (x_n^*, A, B) under the best response, when n is sufficiently large. In other words, $f(0, \delta_n) < 0$.

To show (14), we let $x_n = n^{-\frac{1}{2}} M(\delta_n)$ for a strictly increasing, bounded function $M_n : [0, \epsilon] \to \mathbb{R}_{>0}$ to be specified momentarily. Given the definition of the domain D_n and since M_n is strictly increasing, for any $(x_n, \delta_n) \in D_n$, $\delta_n > 2x_n$. This implies the ordering

$$\frac{1}{2} < q(z_B; \alpha_n) < q(z_A; \alpha_n), \tag{29}$$

given (25) and (26). Moreover,

$$q(z_A; \alpha_n) - q(z_B; \alpha_n) = 3M_n(\delta_n) n^{-\frac{1}{2}} (1 - \delta_n). \tag{30}$$

Consider $p^* \in [0, 1]$ so that

$$p^* r_A = (1 - p^*) r_B. \tag{31}$$

Combining (11) and (30), and using that $\Pr(z_A) r_A > \Pr(z_B) r_B$, for any $\delta_n \in [0, \epsilon]$ we can choose $M_n(\delta_n) > 0$ so that

$$\Pr(z_A | \text{piv}; \alpha) = p^* \tag{32}$$

when n is large enough. Note that, by construction, M_n is indeed an increasing and bounded function. Now, fix δ_n. By construction, given $x_n = M_n(\delta_n) n^{-\frac{1}{2}}$, the voter is indifferent between $(0, A, A)$ and $(0, B, B)$. Since $(0, A, B)$ is a convex combination of $(0, B, B)$ and $(0, A, B)$, the voter is indifferent between $(0, A, B)$ and $(0, A, A)$. However, the voter strictly prefers (x_n^*, A, B) over $(0, A, B)$ and hence over $(0, A, A)$. In other words, $f(M_n(\delta_n) n^{-\frac{1}{2}}, \delta_n) > 0$.

To show (15), we let $\delta_n = n^{-\frac{1}{2}} 3M_n(\epsilon)$. For any $x_n \in \left[0, n^{-\frac{1}{2}} M_n(\delta_n)\right]$, $2x_n \leq \delta_n$, so that

$$|q(z; \alpha_n) - \frac{1}{2}| \leq n^{-\frac{1}{2}} 3M_n(\epsilon). \tag{33}$$

We claim that (33) implies

$$\lim_{n\to\infty} \frac{\Pr(\text{piv}|z;\alpha_n)}{n^{-\frac{1}{2}}} \in \mathbb{R}, \tag{34}$$

To see why, denote $s_n = s(z;\alpha_n)$ and $q_n = q(z;\alpha_n)$. The claim (34) follows from (33) by two observations: First, recall that $((2n+1)s_n)^{-1}$ is the standard deviation of the vote share for A. So, it is given by $s_n = (\frac{(2n+1)}{q_n(1-q_n)})^{-\frac{1}{2}}$ and satisfies

$$\lim_{n\to\infty} \frac{n^{-\frac{1}{2}}}{s_n} \in \mathbb{R} \tag{35}$$

Second, $((2n+1)s_n)^{-1} = \Big[(2n+1)(q_n(1-q_n))\Big]^{-\frac{1}{2}} = s_n(q_n(1-q_n))^{-1}$. Thus,

$$\lim_{n\to\infty} \frac{n^{-\frac{1}{2}}}{((2n+1)s_n)^{-1}} \in \mathbb{R} \tag{36}$$

Given (33), $\lim_{n\to\infty} \delta_n(z;\alpha_n) \in \mathbb{R}$, so that (10) and (36) together imply (34). Consequently,

$$\lim_{n\to\infty} \frac{\text{MB}(x,\delta_n)}{n^{-\frac{1}{2}}} \in \mathbb{R} \tag{37}$$

for $\text{MB}(x_n,\delta_n) = \Big[\Pr(\text{piv}|z_A;\alpha_n,n)q_A r_A + \Pr(\text{piv}|z_B;\alpha_n,n)q_B r_B\Big]$ and all $x_n \in \Big[0, M_n(\delta_n)n^{-\frac{1}{2}}\Big]$.

Now, take any $x''_n \in [0, M_n(\delta_n)n^{-\frac{1}{2}}]$ for which $f(x''_n,\delta_n) = 0$. Denote x'_n the unique precision for which

$$\text{MB}(x''_n,\delta_n) - C'(x'_n) = 0. \tag{38}$$

We claim that

$$\frac{x'_n}{n^{-\frac{1}{2}}} \to \infty \tag{39}$$

as $n \to \infty$. Taylor approximating $C'(x'_n)$ and using that $C'(0) = C''(0) = 0$ gives

$$C'(x'_n) = C'''(\zeta_n)(x'_n)^2 \tag{40}$$

for some $\zeta_n \in [0, x'_n]$. Now, (34) implies

$$\lim_{n\to\infty} \frac{\mathrm{MB}(x''_n, \delta_n)}{n^{-\frac{1}{2}}} \in \mathbb{R}; \tag{41}$$

in particular,

$$\mathrm{MB}(x''_n, \delta_n) \to 0 \tag{42}$$

as $n \to \infty$. Since $C'' > 0$ for $x > 0$ and since $C'(0) = 0$, (38) implies that $x'_n \to 0$ as $n \to \infty$. Further, (40) implies

$$\frac{x'_n}{C'(x'_n)} = \frac{1}{C'''(\zeta_n)x'_n} \to \infty. \tag{43}$$

Finally, we see that (38), (41) imply (39). Therefore, for n large enough, $x'_n > x''_n$, given that $x''_n \leq M_n(\epsilon)n^{-\frac{1}{2}}$ by definition. Since $C'' > 0$, (38) implies

$$\mathrm{MB}(x''_n, \delta_n) - C'(x''_n) > 0, \tag{44}$$

that is, $g(x''_n, \delta_n) > 0$. This means that (15) holds.

To show (16), we let $\delta_n = \epsilon$. This implies that

$$q(z; \alpha_n) > \frac{1}{2} + \frac{\epsilon}{4} \tag{45}$$

when n is large, given (25) and (26). Recalling the Stirling approximation (28) and that the function $q(1-q)$ has the unique maximizer $q = \frac{1}{2}$, the pivotal likelihood is exponentially small in both states, given (45). Now, take any $x''_n \in [0, M_n(\delta_n)n^{-\frac{1}{2}}]$ for which $f(x''_n, \delta) = 0$. The unique precision x'_n for which $\mathrm{MB}(x''_n, \delta_n) - C'(x'_n) = 0$, is also exponentially small. We can write $x'_n = K_n n^{-\frac{1}{2}}$ for some sequence $(K_n)_{n\in\mathbb{N}}$ with $K_n \to 0$ as $n \to \infty$. Recalling the argument that we gave to establish (13), we see that $x''_n > x'_n$ for any $x''_n \in [0, M_n(\delta_n)n^{-\frac{1}{2}}]$ with $f(x''_n, \delta_n) = 0$. Since $C'' > 0$,

$$\mathrm{MB}(x''_n, \delta_n) - C'(x''_n) < 0, \tag{46}$$

that is $g(x''_n, \delta_n) < 0$. In other words, (16) holds.

7.4 Limit outcomes of the inefficient equilibrium sequences.

Here, we give a proof for the statement in the main text that the likelihood that policy A gets elected converges to 1 as $n \to \infty$ given the inefficient equilibrium sequence $(\alpha_n)_{n\in\mathbb{N}}$, given by pairs $(x_n, \delta_n)_{n\in\mathbb{N}}$. Suppose that this is not the case.

Given the vote share ordering (29) and given (9), this implies that

$$\lim_{n\to\infty} \delta_n(z;\alpha_n) \in \mathbb{R} \tag{47}$$

for $z = z_B$. That is, $\delta_n(z;\alpha_n) = n^{-\frac{1}{2}}K_n$ for some sequence $(K_n)_{n\in\mathbb{N}}$ with $\lim_{n\to\infty} K_n \in \mathbb{R}_{>0}$. The same line of argument that we used to establish (39), shows that

$$\lim_{n\to\infty} \frac{x_n}{n^{-\frac{1}{2}}} = \infty. \tag{48}$$

This yields a contradiction since $x_n \leq M_n(\epsilon)n^{-\frac{1}{2}}$ and $M_n(\epsilon)$ is bounded by definition.

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