Persuasion versus Presentation

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Abstract

In many economic situations, people communicate strategically not only to influence the decision-making of their audience but also to shape the perception of certain unobserved characteristics of themselves (e.g. morality, loyalty, or capability). To study such situations, we propose a model of Bayesian persuasion in which a sender endowed with a private type designs the communication about a payoff-relevant state to a receiver. The sender, concerned with both the impacts on the receiver's action and how her type is perceived, aims to strike a balance between persuasion and self-presentation under optimal communication. Whether the receiver fares better or worse compared to the pure persuasion setting may depend on the selected equilibrium, and the welfare effects can be non-monotone with respect to the relative strength of the sender's different motives. We illustrate our findings within various classic payoff environments, for instance with quadratic losses or state-independent sender preferences. Finally, we use the model to shed new light on a wide range of applications.

Keywords: image concerns, persuasion, self-presentation, signaling

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1 Introduction

Many economic situations involve a sender with unobserved characteristics supplying information about a payoff-relevant state to a receiver. Examples are ubiquitous: Voters receive information about the potential impacts of a proposed reform from a politician, whose private gains from the reform's passage are uncertain. Workers learn about the difficulty of a task from a manager, without fully knowing the extent to which their preferences align. Students utilize cheat sheets prepared by their "past-self" to navigate through challenging exam questions, while wondering whether they would have been capable of coming up with the answers instinctively, and so on. In such situations, the receiver may develop beliefs about *both* the state *and* the characteristics of the sender – hereafter referred to as the sender's type – based on the specific information that the sender provides.

In virtually all applications, the sender (she) is motivated to influence the induced belief about the state, typically because that is critical for convincing the receiver (he) to take a favorable action. This "persuasion motive" and its impacts on information revelation have been extensively studied in the economics literature (e.g. Crawford and Sobel, 1982; Grossman, 1981; Kamenica and Gentzkow, 2011; Milgrom, 1981). However, the sender may also have an intrinsic interest in shaping the induced belief about her own type, giving rise to a "presentation motive" in communication. Indeed, this motive is a prominent topic in the social psychology of *self-presentation* since Goffman (1959), which, as summarized by Baumeister (1982), addresses "the use of behavior [...] to please the audience and to construct (create, maintain, and modify) one's public self congruent to one's ideal." For instance, concerns about how one's private traits are perceived come into play when politicians aspire to be seen as non-corrupt in pursuit of electoral support, when managers seek to appear loyal to company values for career advancements, or when individuals fancy themselves as instinctive problem solvers. Despite their prevalence and potential tension with persuasion incentives, such image concerns have received scant attention in previous economic studies on communication.

This paper proposes a model of strategic communication that incorporates both persuasion and presentation motives. In line with the Bayesian persuasion paradigm pioneered by Rayo and Segal (2010) and Kamenica and Gentzkow (2011), we consider a sender who is able to commit to how she will release information about the state before communicating with a receiver. Crucially, our model also endows the sender with a private preference type that differs from the state. This novel feature gives rise to two potentially competing objectives for the sender when designing her communication strategy: persuading the receiver toward favorable actions while presenting herself as a "good" type. We find that the interplay between persuasion and presentation motives can substantially reshape information revelation compared to pure persuasion settings. In particular, whether the sender will release more or less information about the state, and consequently, whether the receiver will fare better or worse compared to the pure persuasion setting, may depend on the selected equilibrium. Additionally, the welfare effects can be non-monotone with respect to the sender's type and the relative strength of her different motives.

As a glimpse into the new insights enabled by our model, consider the motivating binary-state example from Kamenica and Gentzkow (2011), in which a sender attempts to convince a receiver to choose a high action, for instance the adoption of a reform or the conviction of a defendant in court as in the original example, over a low action. Absent persuasive new information, the receiver is inclined to choose the low action. It is known that the sender can maximize the likelihood of successfully persuading the receiver towards the high action through partial disclosure: always revealing the truth in the state where the receiver ranks the high action above the low one but only sometimes so when this is not the case.

Suppose now that the sender has a preference type capturing her private gains from persuading the receiver to take the high action. Such gains may relate to undesirable traits, for instance the corruptness of a politician, or the bias of a prosecutor. The sender desires to be perceived as a "high" type – someone with less private gains, either for instrumental reasons like enhancing electability or accruing credibility in future campaigns/cases, or for purely hedonic reasons such as the gratification of appearing non-corrupt or impartial. Consequently, the sender has an incentive to signal that she is a high type by disclosing information in a way that is costly for lower types to imitate. One intuitive approach to accomplish this is to more often reveal the state when the receiver prefers the low action. This is less attractive for lower types since they have more private gains from influencing the receiver towards the high action. When the sender engages in this type of signaling behavior, it facilitates information revelation and benefits the receiver. However, this approach of separation can work for *all* sender types only if the gain from reputation is not too large. Otherwise, some intermediate type may already be incentivized to furnish the receiver with full information. The highest types can then distinguish themselves only by *withholding* relevant information and revealing the state less often when the receiver ranks the high action above the low one. In other words, an overly strong presentation motive backfires – it leads to less informative communication and welfare losses for the receiver. Ultimately, strong enough image concerns will drive all types to refrain from providing any information to the receiver, as it turns out.

Our general analysis delves into a sender-receiver game with unrestricted state and action spaces, akin to the original work by Kamenica and Gentzkow (2011), and encompassing a continuum of types. The sender, with private knowledge of her preference type, begins by publicly choosing an *information structure* – a joint distribution over states and signals. Following this, the receiver uses the observed signal to deduce the state, and optimizes his choice of action accordingly. In parallel, the receiver also updates his belief about the sender's type based on how the signal was intended to be generated.[1] To capture the different motives of the sender, we posit that her utility function comprises two components: a *material payoff* determined by the state and the receiver's action, and an *image payoff* based on the sender's true type and what type the receiver expects her to be. Further, the sender's image payoff is assumed to increase with her reputation and satisfy the canonical Spence–Mirrlees condition. In our setting, this condition ensures that higher sender types place a greater premium on enhancing their reputation compared to lower types.

As is typical in signaling games, the lack of discipline imposed by Bayes' rule on off-equilibrium beliefs can lead to a large multiplicity of equilibria. Another complicating issue is that, even in pure persuasion settings absent image considerations, pinpointing the optimal information structures for the sender often proves infeasible. To tackle these challenges, we advance in two steps. First, to rule out equilibria that hinge on implausible or unreasonable off-path beliefs of the receiver, we invoke a well-established equilibrium refinement, namely the D1 criterion due to Cho and Kreps (1987) and Banks and Sobel (1987). Second, rather than directly examining the sender's choice of information structures, we adopt a "reduced-form" approach to leverage the key trade-off shaping the sender's equilibrium strategy: her material interests versus image concerns. Specifically, we reformulate the problem as each sender type making choices regarding bundles of an expected material payoff and its associated image payoff, with the constraint that former payoff must be attainable through the use of some information structure. This shift in perspective highlights that the extent to which the sender can mold her image through information design is bounded by the scope of persuasion. Moreover, operating in the payoff space provides tractability – it allows us to identify the

[1] As will become clear through applications, our framework can accommodate situations where the sender's type is also relevant to the receiver's decision-making in a continuation game following the initial interaction.

essential features of D1 equilibria, even when the sender's optimal information structures cannot be expressed in closed form.

Our first result establishes that all equilibria satisfying the D1 criterion manifest as *semi-separating*. Specifically, we find that there is a unique cutoff point, below which all sender types adopt information structures that fully reveal themselves. In contrast, types exceeding this cutoff effectively pool on the same strategy. As the sender's image concern heightens, the cutoff decreases monotonically, resulting in an equilibrium transition from full separation to complete pooling. Furthermore, higher types necessarily earn lower material payoffs in equilibrium. The precise level of material payoff that each below-cutoff type relinquishes for its reputation is pinned down by an envelope formula derived from the local incentive constraints for separation. In tandem, all types above the cutoff obtain the minimum material payoff. Our characterization thus implies that the sender's interim payoffs, both material and image ones, are the same across all equilibria.

Our next results focus on the receiver's welfare, which, as it turns out, can vary substantially across equilibria. This is rarely the case in more conventional settings where the sender signals her private information through channels like education, advertising expenditure, or pricing, as the D1 criterion often identifies a unique equilibrium outcome (Cho and Sobel, 1990; Riley, 2001). In contrast, signaling via information design opens the door to a new potential for indeterminacy: Due to the abundance of information structures, there can be numerous alternatives yielding identical sender payoffs yet significantly different receiver payoffs. Therefore, despite the the sender's payoff-equivalence across all equilibria, whether the receiver's well-being is better or worse compared to the pure persuasion benchmark may depend on the specific equilibrium chosen. Neither the D1 criterion nor any other standard refinements offer guidance in resolving this issue because they rely on discerning unreasonable payoff incentives of the sender. To advance our understanding nonetheless, we provide sufficient conditions under which the welfare consequences of sender's image concerns will be robust or sensitive to equilibrium selection. In doing so, we also identify some general properties of the Pareto frontier within the equilibrium set – specifically, the equilibria maximizing or minimizing the receiver's payoff among all. Most notably, we find that the receiver's expected payoff is necessarily quasi-concave (quasi-convex) – but not always monotonic – with respect to the sender's type in any Pareto-optimal (Pareto-worst) equilibrium. A similar non-monotonicity of the receiver's welfare aslo arises when varying the relative importance of the material and image payoffs in the sender's utility function.

We complement the general analysis of equilibria by specializing the main results in several classic payoff environments, including those with quadratic losses (e.g. Crawford and Sobel, 1982; Melumad and Shibano, 1991) and state-independent sender preferences (e.g. Gentzkow and Kamenica, 2016; Lipnowski and Ravid, 2020). These supplementary exercises offer valuable insights into the nature of information structures emerging in equilibrium, an aspect that our "reduced-form" approach falls short in. In particular, we show that in these commonly studied cases, the Pareto frontier of the equilibrium set can often be supported by simple families of information structures, such as censorship, interval disclosure, or a mix between full revelation and total secrecy.

In the last part of the paper, we apply our theory to three different contexts. In each instance, we provide a tangible interpretation of the sender's type and elucidate the origin of image concerns. Our first application considers a self-signaling environment (Bodner and Prelec, 2003) where the sender and receiver represent two selves of the same agent at different points in time. The application is cast in the context of a mental task; for example, an exam or a strategic choice. We show that self-image concerns, such as pride, can prompt self-handicapping, wherein the agent abstains from acquiring information that could otherwise aid in solving the mental task. This form of information avoidance is well-documented empirically (see, e.g., the survey by Golman, Hagmann and Loewenstein, 2017), and existing literature has offered theoretical explanations for similar behavior tied to image concerns in other contexts (e.g. Bénabou and Tirole, 2002; Grossman and Van der Weele, 2017). The innovation here lies in our consideration of a setting where the sender's scope of information acquisition is unrestricted, thereby reinforcing and extending previous arguments.

Next, we apply our theory to the realm of organizational economics, focusing on a moral-hazard situation where a manager controls the flow of information accessible to a worker. The manager privately knows the extent to which her preferences concerning the worker's efforts align with the company's leadership, as opposed to the worker. Further, the manager aspires to project an image of compliance with the company's leadership, recognizing its positive impact on her career prospects. We find that the manager may choose to hide information from the worker in an effort to impress superiors, even if it is potentially detrimental to the company's interest. Our result complements Jehiel (2015)'s insights on the drivers of intransparency within organizations, shedding light on why many companies nowadays move away from performance reviews done solely by (direct) superiors to employing committees that involve third persons.

Finally, we present an application to political economy, inspired by the seminal work of Fernandez and Rodrik (1991) on policy stagnation. We contemplate a scenario where a politician conveys information about a reform to a group of voters. The politician, whose personal interest in the reform remains concealed from the voters, faces a trade-off: providing information skewed in favor of the reform increases its chance of being accepted but may be seen as self-serving. Such a perception will adversely affect the politician's electability in future campaigns, as voters prefer leaders who act in the public's best interest rather than their own. Our analysis suggests that in situations where future electoral outcomes hinge heavily on the perceived morality of the politician, she may opt to endorse studies that consistently align with voters' prior skepticism toward the reform, thereby perpetuating their ignorance and causing policy stagnation.

Related literature. Our paper primarily contributes to the burgeoning literature of Bayesian persuasion and information design. For an excellent overview of this literature, see Bergemann and Morris (2019) and Kamenica (2019). What sets our paper apart is the introduction of a novel dimension alongside the conventional persuasion motive, namely the presentation motive of the sender. Our general model remains agnostic about the origins of this motive. It could capture the instrumental benefits that individuals gain from their reputation in future interactions (Morris, 2001; Sobel, 1985), or stem from a wide array of psychological preferences (Geanakoplos, Pearce and Stacchetti, 1989), such as conformity (Bernheim, 1994), social esteem (Bénabou and Tirole, 2006), or self-image concerns (Baumeister, 1998; Bodner and Prelec, 2003; Köszegi, 2006). Regardless of its source, the critical implication of incorporating this motive into our model is that the sender's communication strategy will inherently reflect what she privately knows: her own preference type. Several papers have investigated how private sender information may influence persuasion, e.g. Chen and Zhang (2020); Degan and Li (2021); Hedlund (2017); Koessler and Skreta (2023); Perez-Richet (2014). However, in the existing work the sender holds private information directly related to a relevant state that she designs information over. In contrast, in our model, the sender's private information captures aspects of her preferences that are independent of the state, and this private information matters due to the sender's image concerns. We argue that such intrinsic preference traits can only be revealed through signaling rather than through an information structure designed over them. More broadly, our work is complementary to a set of recent studies that explore Bayesian persuasion with similar features on the receiver's side, including reputational concerns (Li, 2022; Salas, 2019), psychological preferences (Lipnowski and Mathevet, 2018; Schweizer and Szech, 2018), and private information (Guo and Shmaya, 2019; Hu and Weng, 2021; Kolotilin, Mylovanov, Zapechelnyuk and Li, 2017) of the receiver.

We also make a contribution to the classic literature on signaling games following Spence (1973)'s seminal work. The semi-separating equilibrium structure, a key feature of our model, has been observed in other signaling contexts in the past (e.g., Bernheim, 1994; Cho and Sobel, 1990; Kartik, 2009). In these earlier studies, the incomplete separation of types is mainly driven by exogenous constraints on the signal space available to the sender. In contrast, this phenomenon emerges in our model due to a boundary on the sender's payoff that is endogenously determined by the scope of persuasion. More substantially, existing research has primarily treated signaling as a means to influence the receiver's action – that is to persuade the receiver in our terminology – whereas our paper takes on scenarios in which a strategic tension arises between signaling and persuasion. This alternative approach that we propose not only opens up new applications but also yields theoretical implications differing from prior works, especially regarding the receiver's welfare.[2]

Finally, our paper complements the literature examining reputation-building behavior in repeated interactions (Mailath and Samuelson, 2006, 2015). Within this literature, studies have identified various scenarios where reputational concerns can lead to advantageous outcomes (e.g. Fudenberg and Levine, 1989) or adverse effects (e.g. Ely, Fudenberg and Levine, 2008; Ely and Välimäki, 2003) in terms of welfare consequences. Results from these studies typically involve a rational player (the "normal" type) being motivated to emulate the behavior of a non-strategic player who adheres to an exogenous decision rule (the "behavioral" or "commitment" type). In our setting, whether reputation has positive, negative, or ambiguous effects on the receiver's welfare depends on various details of the game, such as the alignment of the players' material interests and the distribution of the sender's type.

[2] A strand of the signaling literature has considered the role of image concerns in cheap-talk communication. However, many of these studies tend to tackle more specialized questions compared to ours, such as whether a desire to influence the receiver's future decisions (Morris, 2001; Sobel, 1985) or to appear well-informed (Ottaviani and Sørensen, 2006a,b) would encourage or discourage honest communication. Others focus on separate themes, e.g. the potential of third-party bribes to enhance information transmission when the sender likes to be perceived as non-corruptible (Durbin and Iyer, 2009).

The remainder of the paper is organized as follows:Section 2 introduces the formal model. Section 3 presents the main theoretical results characterizing the equilibria and welfare outcomes, accompanied by specific examples in classic payoff environments. Section 4 details the applications of our theory. Finally, Section 5 concludes. Technical proofs and analytical details that support the main text are provided in the Appendix.

2 Model

We study a communication game between a sender (she) and a receiver (he). There is a state space Ω, with a typical state denoted by ω, and an action space A, with a typical action denoted by a. Both A and Ω are compact metric spaces. The players are uncertain about the state at the outset of the game, but they have a common prior $\mu_0 \in \Delta(\Omega)$ about it with full support. The sender moves first by choosing an information structure $\pi \in \Delta(\Omega \times A)$, which is a joint distribution of the state and a signal with marginal μ_0. The set of all such distributions is represented by Π. Given that the signal space is contained in the action space, each signal realization s can be considered as an action recommendation from the sender to the receiver. The receiver observes the sender's choice of information structure and the signal realization, and finally chooses an action $a \in A$.

Preferences. The receiver has a continuous utility function $u(a, \omega)$ that depends on both his action and the state of the world. The sender is endowed with a private type $\theta \in \Theta \equiv [0, 1]$, which is commonly known to be distributed according to an absolutely continuous distribution function with full support. The sender's utility is the sum of two continuous function: $v(a, \omega) + \phi \cdot w(p(\eta), \theta)$, where $\eta \in \Delta(\Theta)$ denotes the receiver's belief about the sender's type, and $p(\eta) \equiv \mathbb{E}_\eta[\tilde{\theta}]$ is interpreted as the sender's *image.* Naturally, $\phi > 0$ measures how much the sender cares about the image payoff $w(p, \theta)$ relative to the material payoff $v(a, \omega)$. Further, the function $w(\cdot)$ is continuously differentiable and adheres to the following conditions:

$$\frac{\partial w(p,\theta)}{\partial p} > 0 \text{ and } \frac{\partial^2 w(p,\theta)}{\partial p \partial \theta} > 0. \tag{1}$$

In words, the first part expressed in (1) indicates that all sender types prefer to be perceived as high types. The second part, akin to the well-known Spence-Mirrlees (or increasing difference) condition, asserts that this desire is stronger for higher types.

Strategies and equilibrium. A pure strategy of the sender is a mapping $\sigma : \Theta \to \Pi$ that specifies for each type an information structure. A pure strategy of the receiver is a mapping that specifies an action for every possible information structure and signal realization. We analyze the perfect Bayesian equilibria in pure strategies (Fudenberg and Tirole, 1991, p. 333; henceforth equilibrium). Following this equilibrium concept, the receiver, based on the sender's choice of information structure and the realized signal, forms posterior beliefs about the state using Bayes' rule. Subsequently, he selects an action among those that maximize his expected utility. We assume that whenever the receiver is indifferent between multiple actions and one of them is recommended by the sender, he will choose that action. The receiver also updates his beliefs about the sender's type, taking into account what information the sender opted to disclose or withhold. Consequently, the sender's strategy influences not only the material outcome of the game but also her image in the eyes of the receiver.[3]

In our setting, a revelation principle holds. For any equilibrium, there exists an equivalent equilibrium in which the receiver consistently obeys the sender's recommendation; that is, he chooses $a = s$ after whenever a signal $s \in A$ is realized. Specifically, for every sender type $\theta \in \Theta$, the two equilibria will result in the same joint distribution over the state and receiver action, as well as the same type perception by the receiver. The proof is relegated to the Appendix A.1. In that section, we also demonstrate two additional points. Firstly, it is not necessary to consider signal spaces that go beyond the receiver's action space. Secondly, our chosen tie-breaking rule on the receiver's side is without loss for the characterization of outcomes that occur on the equilibriun path, given a mild condition on the players' payoff functions satisfied in all examples and applications in this paper.[4] Based on this revelation principle, we can identify an equilibrium with an *incentive compatible* sender strategy $\sigma = \{\pi_\theta\}_{\theta\in\Theta}$ and a belief system $H = \{\eta(\pi)\}_{\pi\in\Pi}$ such that each $\eta(\pi) \in \Delta(\Theta)$ is consistent with Bayes' rule given σ. Here, incentive compatibility requires that for every $\theta \in \Theta$, the associated

[3] If the sender could design an information structure over both ω and θ, and commit to it before learning θ, our model would become a special case of Kamenica and Gentzkow (2011). Alternatively, if the design over ω and θ occurred after the sender gets to know θ, the setting would be similar to Koessler and Skreta (2023). We refrain from an extensive investigation of these alternative specifications in our context, because information design about the strength of image concerns seems difficult to justify in practice.

[4] The core implication of this condition is that information can be used to arbitrarily closely replace the tie-breaking rule. Relatedly, Lipnowski, Ravid and Shishkin (2023) provide various conditions for the tie-breaking assumption to be insubstantial in standard Bayesian persuasion games without a signaling component.

information structure π_θ is a solution to

$$\max_{\pi \in \Pi^*} \mathbb{E}_\pi[v(s,\omega)] + \phi \cdot w(p(\eta(\pi)),\theta), \tag{2}$$

where

$$\Pi^* \equiv \left\{ \pi \in \Pi : s \in \arg\max_{a \in A} \mathbb{E}\left[u(a,\omega)|s;\pi\right] \;\; \forall s \in supp(\pi) \right\}. \tag{3}$$

In words, given the receiver's system of beliefs and the constraint that following the sender's recommendation is indeed optimal for the receiver, no sender type can be strictly better off by deviating from the strategy σ.[5]

Equilibrium refinement. Since Bayes' rule does not put any restriction on the receiver's out-of-equilibrium beliefs about the sender's type, the usual equilibrium multiplicity of signaling games also arises in our model. We follow the literature and invoke a standard equilibrium refinement, the D1 criterion due to Cho and Kreps (1987) and Banks and Sobel (1987). The core idea is to restrict the receiver's out-of-equilibrium beliefs to the sender types that are "most likely" to benefit from deviations to off-path choices. Specifically, the D1 criterion requires that if, for a type θ, there is another type θ' that has a strict incentive to deviate to the off-path choice $\pi \in \Pi^*$ whenever θ has a weak incentive to do so, then the receiver's out-of-equilibrium beliefs upon observing this choice of the sender shall not put any weight on θ (see Appendix A.2 for the formal statements). An equilibrium that passes this test is a *D1 equilibrium*; henceforth, often simply called equilibrium if no misunderstanding is possible.

3 Analysis

3.1 A Reduced-Form Characterization of Equilibria

Kamenica and Gentzkow (2011) analyze the benchmark scenario in which the sender does not have image concerns ($\phi = 0$), that is, she is purely guided by the persuasion motive. It is known that, even in that setting, the equilibrium information structure is often intractable.

[5] Under the tie-breaking rule that we imposed, restricting the sender's choice to the set $\Pi^* \subsetneq \Pi$ is without loss for characterizing the equilibria. This is because we can further complete the belief system $\eta(\cdot)$ for information structures $\pi \notin \Pi^*$ in a way that any choice of such information structures would be inferior compared to $\sigma(\theta)$ for all sender types $\theta \in \Theta$. See Appendix A.1 for a formal argument.

This problem does not get any easier, if not more difficult, in our model, because the sender's persuasion motive is entangled with her presentation motive. To make progress, we simplify the infinite-dimensional maximization problem (2) of the sender by moving the analysis to the interim stage. In particular, instead of analyzing information structures directly, we focus on the expected material payoff that the sender obtains by the choice of an information structure. Similar "reduced-form approaches" have proven useful in a variety of mechanism design settings (e.g., Ben-Porath, Dekel and Lipman, 2014; Che, Kim and Mierendorff, 2013). Below, we demonstrate the power of such an approach in the context of signaling through the design of information structure.

We start by observing that, when viewing the game at the interim stage, it exhibits a number of useful properties. First, the interim game is monotonic in the sense of Cho and Sobel (1990), because, holding the expected material payoff fixed, all sender types share the same ordinal preferences over their images in the eyes of the receiver.[6] Second, the set of (expected) material payoffs that the sender can implement through her choice of information structure is a compact interval. To see this, consider the payoffs

$$\bar{V} \equiv \max_{\pi \in \Pi^*} \mathbb{E}_\pi[v(s,\omega)] \quad \text{and} \quad \underline{V} \equiv \min_{\pi \in \Pi^*} \mathbb{E}_\pi[v(s,\omega)], \tag{4}$$

and let $\bar{\pi}$ and $\underline{\pi}$ be two information structures that give rise to $\bar{V}$ and $\underline{V}$, respectively.[7] Any implementable material payoff must be weakly larger than $\underline{V}$ and weakly smaller than $\bar{V}$. Conversely, any material payoff in between can be achieved by appropriately mixing the information structures $\bar{\pi}$ and $\underline{\pi}$.[8] Hence, the set of implementable material payoffs is exactly $[\underline{V}, \bar{V}]$. Third, as we formally show in the Appendix (Lemma A1), the sender types' preferences display the following single-crossing property in the interim space: for any two bundles (V,η), $(V',\eta') \in [\underline{V},\bar{V}] \times \Delta(\Theta)$ with $V < V'$, if type θ weakly prefers (V,η) over (V',η'), then all types $\theta' > \theta$ will strictly prefer (V,η) over (V',η').

The above properties allow us to apply techniques from the costly signaling literature (e.g. Cho and Sobel, 1990; Mailath, 1987; Ramey, 1996) to partially characterize the set of D1

[6] This property implies that our equilibrium selection is robust to alternative criteria such as Universal Divinity (Banks and Sobel, 1987) and Never-a-Weak-Best-Response (Kohlberg and Mertens, 1986), as they are equivalent to D1 in monotonic games (see Proposition 1 of Cho and Sobel, 1990).

[7] Both $\bar{\pi}$ and $\underline{\pi}$ exist because v is continuous and Π^* is compact with respect to the weak-* topology.

[8] To implement the payoff $V = \lambda \underline{V} + (1-\lambda)\bar{V}$ for some $\lambda \in [0,1]$, we may use the following "grand" information structure $\hat{\pi}$: with probability λ, the sender draws a signal s according to $\underline{\pi}$, and with probability $1-\lambda$, according to $\bar{\pi}$. It can be checked that $\hat{\pi} \in \Pi^*$ and $\mathbb{E}_{\hat{\pi}}[v(s,\omega)] = V$, i.e., $\hat{\pi}$ indeed implements V.

equilibria. Given a sender strategy σ, we define $V(\theta;\sigma) \equiv \mathbb{E}_{\pi_\theta}[v(s,\omega)]$ and $p(\theta;\sigma) \equiv \mathbb{E}[\tilde{\theta}|\tilde{\theta} : \sigma(\tilde{\theta}) = \sigma(\theta)]$, i.e., the expected material payoff and the perceived image that the strategy induces for each type θ, respectively. We say that a type θ is *separating* under the strategy σ if $\sigma(\theta') \neq \sigma(\theta)$ for all $\theta' \neq \theta$ (in which case we necessarily have $p(\theta;\sigma) = \theta$). Otherwise, we say that θ is *pooling.* Our first result shows that there exists a *unique* cutoff $\hat{\theta}$ such that all types $\theta < \hat{\theta}$ ($\theta \geq \hat{\theta}$) will be separating (pooling) in any equilibrium that satisfies D1.[9] Moreover, although the D1 criterion may not select a unique equilibrium, it fully pins down the equilibrium payoff of the sender.

Theorem 1. *There is a unique cutoff* $\hat{\theta} \in [0,1) \cup +\infty$ *such that any strategy* $\sigma = \{\pi_\theta\}_{\theta\in\Theta}$ *of the sender with* $\pi_\theta \in \Pi^*$ *for all* $\theta \in [0,1]$ *is part of a D1 equilibrium if and only if the following two conditions are both satisfied:*

(i) All types $\theta < \hat{\theta}$ *are separating, with*

$$V(\theta;\sigma) = \bar{V} - \phi \cdot \int_0^\theta \frac{\partial w(x,x)}{\partial p} dx; \tag{5}$$

(ii) All types $\theta \geq \hat{\theta}$ *are pooling, with* $V(\theta;\sigma) = \underline{V}$ *and* $p(\theta;\sigma) = \mathbb{E}[\tilde{\theta}|\tilde{\theta} \geq \hat{\theta}]$.

Theorem 1 implies the existence of a D1 equilibrium — one can always construct a strategy that satisfies (i) and (ii) (recall that the set of implementable material payoffs is $[\underline{V}, \bar{V}]$). Exactly which D1 equilibrium is chosen is immaterial for the sender, because from her perspective all of them are equivalent in terms of payoffs. Therefore, the set of D1 equilibria can be Pareto-ranked according to the welfare of the receiver. In later analysis, we will provide examples and applications which also feature payoff equivalence for the receiver, or which permit an analytic description of the equilibria that are extremal in the Pareto ranking.

In what follows, we prove the only-if part of Theorem 1, i.e., that all D1 equilibria necessarily satisfy conditions (i) and (ii), which is instructive as it highlights how the equilibrium outcome is shaped by the tension between the conflicting motives of the sender. The proof of the if-part of the theorem, i.e., that all strategies satisfying (i) and (ii) are part of a D1 equilibrium, is relegated to the appendix as it is rather mechanical: Types would not want to

[9] We assume that if a type (e.g., the cut-off type $\hat{\theta}$ when $\hat{\theta} \in (0,1)$) is indifferent between separating herself or pooling with some higher types, she would break the tie in favor of the latter. With a continuous type distribution, this tie-breaking rule is inconsequential.

mimic each other because conditions (i) and (ii) will be derived (among others) from the on-path incentive compatibility constraints. With attention to detail, one can further construct the appropriate out-of-equilibrium beliefs that prevent off-path deviations and satisfy D1.

Monotone strategies and incomplete separation. To begin, we establish some qualitative features of the sender's strategy based on her equilibrium incentives. Recall that the sender's central trade-off is between the material benefits derived from persuasion and the reputational gains achieved through presentation. In particular, the sender is willing to sacrifice her material payoff only if doing so results in a more favorable image. Further, since the image payoff function $w(\cdot)$ satisfies the Spence-Mirrlees (or increasing difference) condition in (1), this kind of "money-burning" incentive is strictly higher for higher types. Lemma 1 below exploits this property and shows that any equilibrium must be monotone in the sense that the interim material payoff of higher types is lower, while their image is higher.

Lemma 1. *In any equilibrium, $V(\theta;\sigma)$ is decreasing in θ and $p(\theta;\sigma)$ is increasing in θ.*

Next, we show that a type cannot be pooling unless she receives the lowest possible material payoff among the implementable ones.

Lemma 2. *In any equilibrium that satisfies the D1 criterion, $\forall\theta \neq \theta'$, if $\sigma(\theta) = \sigma(\theta')$, then $V(\theta;\sigma) = V(\theta';\sigma) = \underline{V}$.*

The intuition behind Lemma 2 is as follows. Given the single-crossing property of the sender's interim preferences, a higher type in a pooling set will be more likely to benefit from an off-path choice that slightly reduces her material payoff than any type lower than her. To be consistent with the D1 criterion, such an unexpected move must convince the receiver that the sender's type is weakly higher than anyone in that pooling set. As a consequence, a pooling type can obtain a discrete gain in image payoff by sacrificing an arbitrarily small amount of material payoff. This kind of deviation is not a threat to the equilibrium if and only if the material payoff is already "used up" by the pooling types: they are receiving $\underline{V}$, the lowest possible material payoff, so undercutting is simply not feasible.

Lemmas 1 and 2 jointly imply that, in any D1 equilibrium, the sender must use a strategy where all types below a cutoff $\hat{\theta}$ separate by monotonically decreasing their material payoff, while all types above $\hat{\theta}$ cluster at the lower boundary of the material-payoff range. Similar incomplete separation at the top has been established in other contexts (e.g. Bernheim, 1994; Kartik, 2009). Cho and Sobel (1990) demonstrated that this semi-separating structure is inherent to the equilibria selected by D1 in a broad class of costly signaling games with a compact interval of signals available to the sender. The key distinction is that, in our framework, the relevant costs or boundaries are not exogenously imposed on the signals; rather, they arise endogenously from how the receiver reacts under varying information structures.

The cost of reputation. We now proceed to characterize the intensity of signaling for types in the separating interval $[0, \hat{\theta})$; that is, how much material payoff will be sacrificed by such types. Here, the central idea is to leverage that the sender's utility function is quasi-linear with respect to her image payoff. This payoff structure reminds of the standard mechanism design setting with transfers. Thus, we advance the analysis by applying the classical envelope theorem argument (see e.g. Proposition 23.D.2 in Mas-Colell, Whinston and Green, 1995) to the local incentive compatibility constraints of the sender.

Take any $\theta \in [0, \hat{\theta})$. Note that for sufficiently small $\epsilon > 0$, we have $\theta + \epsilon \in [0, \hat{\theta})$ as well. Incentive compatibility for the type-θ sender implies the following:

$$\phi \cdot [w(\theta + \epsilon, \theta) - w(\theta, \theta)] \leq V(\theta; \sigma) - V(\theta + \epsilon; \sigma). \tag{6}$$

That is, the image gain for type θ from mimicking $\theta + \epsilon$ is weakly smaller than the associated loss in material utility. Similarly, incentive compatibility for type $\theta + \epsilon$ implies:

$$\phi \cdot [w(\theta + \epsilon, \theta + \epsilon) - w(\theta, \theta + \epsilon)] \geq V(\theta; \sigma) - V(\theta + \epsilon; \sigma). \tag{7}$$

Combining (6) and (7) and dividing them by ϵ, we have

$$\frac{\phi \cdot [w(\theta + \epsilon, \theta) - w(\theta, \theta)]}{\epsilon} \leq \frac{V(\theta; \sigma) - V(\theta + \epsilon; \sigma)}{\epsilon} \leq \frac{\phi \cdot [w(\theta + \epsilon, \theta + \epsilon) - w(\theta, \theta + \epsilon)]}{\epsilon}.$$

Since $w(\cdot)$ is continuously differentiable, it follows from the squeeze theorem that

$$V'(\theta; \sigma) \equiv \lim_{\epsilon \to 0} \frac{V(\theta + \epsilon; \sigma) - V(\theta; \sigma)}{\epsilon} = -\phi \cdot \frac{\partial w(\theta, \theta)}{\partial p}. \tag{8}$$

Hence, $V(\cdot; \sigma)$ is also continuously differentiable.

Further, whenever $\hat{\theta} > 0$, the type $\theta = 0$ is in the separating interval and gets the lowest possible image payoff. Thus, incentive compatibility also requires that this type must be earning the highest possible material payoff, i.e., $V(0;\sigma) = \bar{V}$. By combining this boundary condition with the differential equation (8), we immediately obtain the payoff formula (5) and conclude that it must hold for all $\theta \in [0, \hat{\theta})$ in any D1 equilibrium.

Uniqueness of the equilibrium cutoff. To complete the proof of Theorem 1, it remains to show that the cutoff $\hat{\theta}$ is unique across all D1 equilibria. The characterization of the equilibrium payoffs on $[0, \hat{\theta})$ implies that the following indifference condition must hold for an *interior* cutoff type $\hat{\theta} \in (0, 1)$:

$$\left(\bar{V} - \phi \cdot \int_0^{\hat{\theta}} \frac{\partial w(x,x)}{\partial p} dx\right) + \phi \cdot w(\hat{\theta}, \hat{\theta}) = \underline{V} + \phi \cdot w\left(\mathbb{E}[\tilde{\theta}|\tilde{\theta} > \hat{\theta}], \hat{\theta}\right). \tag{9}$$

Intuitively, if condition (9) does not hold, then, by continuity either some pooling type $\hat{\theta} + \epsilon$ would have a strict incentive to mimic, e.g., the separating type $\hat{\theta} - \epsilon$, where $\epsilon > 0$ is sufficiently small, or vice versa. We rewrite (9) as $(\bar{V} - \underline{V})/\phi = I(\hat{\theta})$, where the mapping $I(\cdot)$ is given by

$$I(\theta) = \int_0^{\theta} \frac{\partial w(x,x)}{\partial p} dx \; + w(\mathbb{E}[\tilde{\theta}|\tilde{\theta} > \theta], \theta) - w(\theta, \theta),$$

for all $\theta \in [0, 1]$. Note that $I(\cdot)$ is strictly increasing.[10] Also, $I(\cdot)$ is continuous because $w(\cdot)$ is continuously differentiable and the type distribution is absolutely continuous.

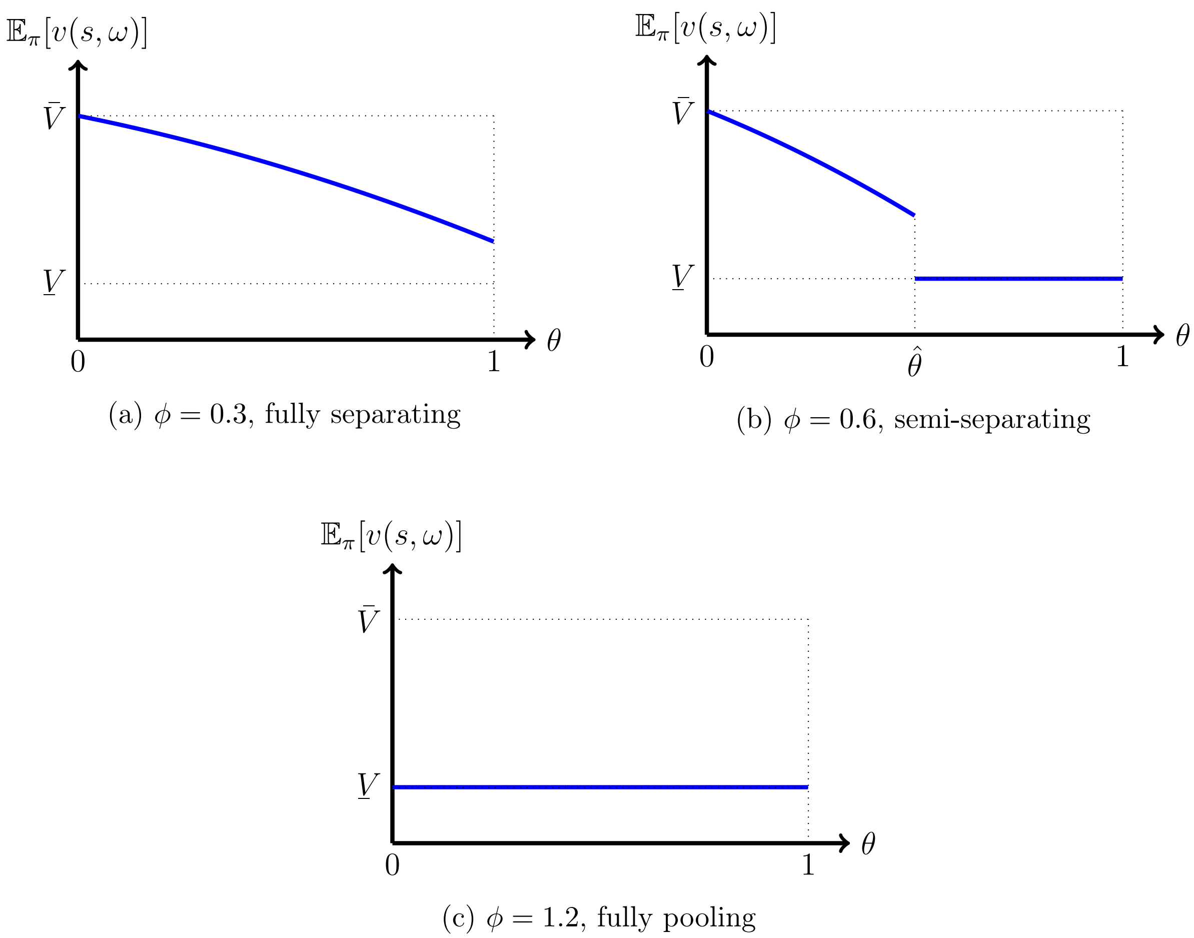
We distinguish three cases. First, if $I(0) < (\bar{V} - \underline{V})/\phi < I(1)$, the intermediate value theorem assures that $(\bar{V} - \underline{V})/\phi = I(\hat{\theta})$ admits an interior solution $\hat{\theta} \in (0, 1)$, and this solution is unique due to the strict monotonicity of $I(\cdot)$.

Second, consider the case $(\bar{V} - \underline{V})/\phi \geq I(1)$ or, equivalently, $\phi \leq \underline{\phi} \equiv (\bar{V} - \underline{V})/I(1)$.

[10] For all $\theta, \theta' \in [0, 1]$ with $\theta' < \theta$, we have

$$\begin{aligned} I(\theta) - I(\theta') &= \int_{\theta'}^{\theta} \frac{\partial w(x,x)}{\partial p} dx + \int_{\theta}^{\mathbb{E}[\tilde{\theta}|\tilde{\theta}>\theta]} \frac{\partial w(x,\theta)}{\partial p} dx - \int_{\theta'}^{\mathbb{E}[\tilde{\theta}|\tilde{\theta}>\theta']} \frac{\partial w(x,\theta')}{\partial p} dx \\ &> \int_{\theta'}^{\theta} \frac{\partial w(x,\theta')}{\partial p} dx + \int_{\theta}^{\mathbb{E}[\tilde{\theta}|\tilde{\theta}>\theta]} \frac{\partial w(x,\theta')}{\partial p} dx - \int_{\theta'}^{\mathbb{E}[\tilde{\theta}|\tilde{\theta}>\theta']} \frac{\partial w(x,\theta')}{\partial p} dx \\ &= \int_{\mathbb{E}[\tilde{\theta}|\tilde{\theta}>\theta']}^{\mathbb{E}[\tilde{\theta}|\tilde{\theta}>\theta]} \frac{\partial w(x,\theta')}{\partial p} dx \\ &\geq 0, \end{aligned}$$

where the strict inequality follows since $w(\cdot)$ has strictly increasing differences, and the weak inequality holds because $w(p, \theta')$ is strictly increasing in p and $\mathbb{E}[\tilde{\theta}|\tilde{\theta} > \theta] \geq \mathbb{E}[\tilde{\theta}|\tilde{\theta} > \theta']$.



(a) $\phi = 0.3$, fully separating

(b) $\phi = 0.6$, semi-separating

(c) $\phi = 1.2$, fully pooling

Figure 1: Sender's expected material payoff as a function of her type in a D1 equilibrium, with $\theta \sim \mathcal{U}[0,1]$, $w(p,\theta) = p \cdot (\theta + 1)$, $\bar{V} - \underline{V} = 0.6$, and different ϕ.

Suppose that there would be an equilibrium with cutoff $\hat{\theta} < 1$. Then, all types $\theta < 1$ would strictly prefer separating over pooling with higher types, contradicting the assumption of $\hat{\theta} < 1$. As a result, any equilibrium selected by D1 must be fully separating and we can write $\hat{\theta} = +\infty$ without loss of generality.

Third, consider the case when $(\bar{V} - \underline{V})/\phi \leq I(0)$ or, equivalently, $\phi \geq \bar{\phi} \equiv (\bar{V} - \underline{V})/I(0)$. Suppose that there would be an equilibrium with cutoff $\hat{\theta} > 0$. Then, all types $\theta < 1$ would strictly prefer pooling with higher types over separation (except for type 0, who may be indifferent), which contradicts the assumption of $\hat{\theta} > 0$. This implies that all types must be pooling in any equilibrium, and consequently, we have $\hat{\theta} = 0$ as the unique cutoff. □

We close this section with a visual representation of the main findings of Theorem 1. Figure 1 presents all three types of sender's strategy that could emerge in an equilibrium satisfying the D1 criterion: strategies that lead to full separation (Panel a), semi-separation (Panel b), and complete pooling (Panel c).

3.2 Pareto (In)efficiency and Equilibrium Multiplicity

As previously discussed, Theorem 1 reveals that the sender's interim payoffs are equivalent across all D1 equilibria. In particular, the theorem describes precisely the level of material payoff that each sender type will give up in order to separate herself from lower types. Given the abundance of possible information structures, there are, however, manifold ways how types can make such sacrifices. In other words, Theorem 1 does not give a very sharp prediction regarding the specific information structure that the sender will adopt, which also means that the receiver's payoff may not be definitely pinned down. This characteristic underscores the distinction of signaling through information design compared to more conventional settings, such as those where senders employ education, advertising, or pricing as signals, as the D1 criterion typically identifies a unique equilibrium outcome in those instances (see e.g. Cho and Sobel, 1990; Riley, 2001).

Since there is no a priori reason to restrict attention to a specific class of information structures, we pursue three avenues in the following:: (i) establishing simple sufficient conditions under which the implications of the sender's image concerns for receiver welfare will be robust or sensitive to equilibrium selection, (ii) analyzing the Pareto-frontier of the equilibrium set, and (iii) applying these overarching findings to various specialized yet classic payoff environments (e.g., quadratic losses or state-independent sender preferences). To simplify the discussion, we make two additional mild assumptions. First, information is valuable to the receiver, in the sense that his expected payoff under full information is strictly higher than that under no information: $\bar{U} \equiv \mathbb{E}_{\mu_0}\left[\max_{a \in A} u(a,\omega)\right] > \underline{U} \equiv \max_{a \in A} \mathbb{E}_{\mu_0}\left[u(a,\omega)\right]$. Second, in the benchmark scenario in which the sender has *no* image concerns, the receiver's equilibrium payoff – which we denote by U^* – is uniquely defined.[11]

3.2.1 When will sender's image concerns be unequivocally harmful?

When does the presence of sender's image concerns harms the receiver's welfare, irrespective of which D1 equilibrium is selected? A straightforward sufficient condition is that the receiver would earn his full-information payoff when the sender does not have any image concern. Our next result summarizes this simple observation and goes beyond it by describing the properties

[11] Formally, the second assumption requires that $\mathbb{E}_{\pi}[u(a,\omega)] = \mathbb{E}_{\pi'}[u(a,\omega)]$ for all $\pi, \pi' \in \Pi^*$ that maximize the sender's material payoff. Without this assumption, the analysis in this section still applies if we adjust the benchmark referred to in each result accordingly to the maximum or minimum utility attainable by the receiver under any information structure that maximizes the sender's material payoff.

of the best- and worst-case scenarios for the receiver: the *Pareto-optimal* and *Pareto-worst* D1 equilibria, respectively. Formally, we say that a D1 equilibrium is Pareto-optimal if the sender strategy σ associated with this equilibrium maximizes the receiver's payoffs type-wise across all sender strategies consistent with Theorem 1's characterization; that is,

$$\sigma(\theta) \in \arg \max_{\pi \in \Pi^* : \mathbb{E}_\pi[v(s,\omega)] = V(\theta;\sigma)} \mathbb{E}_\pi[u(s,\omega)]. \tag{10}$$

for all $\theta \in \Theta$. The Pareto-worst D1 equilibria are defined analogously.[12]

Theorem 2. *If $U^* = \bar{U}$, the receiver can never benefit from the presence of the sender's image concerns. Moreover,*

(i) provided that $\phi < \bar{\phi}$ (so that we have a cutoff type $\hat{\theta} > 0$), there exists a D1 equilibrium in which the receiver is strictly worse off compared to the case without image concerns;[13]

(ii) in any Pareto-optimal D1 equilibrium, the receiver's expected payoff is decreasing with respect to the sender's type;

(iii) in any Pareto-worst D1 equilibrium, the receiver's expected payoff is quasi-convex with respect to the sender's type.

Intuitively, the conditions of Theorem 2 imply that a no-disclosure protocol is suboptimal for the sender when she is purely guided by material interests, since otherwise the receiver would not have been able to enjoy his full-information payoff. Therefore, an image-concerned sender can always separate herself from those very low types by occasionally sending a completely uninformative signal to the receiver, which obviously engenders a negative "side-effect" on the receiver's payoff. As for the properties of the Pareto-extremal equilibria, our proof mainly exploits the convexity of the set of payoff profiles that can be implemented via information design: For instance, suppose, within the separating interval of an equilibrium, the receiver's payoff implied by the strategy of a type θ is lower than that of a higher type $\theta' > \theta$. This equilibrium cannot be Pareto-optimal, for the following reason: Replacing the information structure that type θ initially chooses with an appropriate mix of those used by types

[12] An alternative definition would be to Pareto-rank equilibria based on the *ex-ante* expected utility of the receiver. Under this definition, multiple equilibria that only differ on a null set of types can all be Pareto-optimal/worst, but the characterization of the Pareto-frontier remains unchanged in all other aspects.

[13] The statement remains valid when $\phi \geq \bar{\phi}$ under the additional assumption that an information structure $\pi \in \Pi^*$ satisfying $\mathbb{E}_\pi[v(s,\omega)] = \underline{V}$ and $\mathbb{E}_\pi[u(s,\omega)] < \bar{U}$ exists.

0 and θ' will not change the sender's payoff, but will strictly improve the receiver's payoff. A similar but slightly more intricate constructive argument (which involves the no-disclosure protocol instead of the one used by type 0) shows that any Pareto-worst equilibrium must be either decreasing or U-shaped with respect to the sender's type. Otherwise, it would have been feasible to further reduce the receiver's payoff without altering the sender's.

In what follows, we exemplify the insights of Theorem 2 within various classic settings from the literature on sender-receiver games.

Example 1: Congruent preferences. Suppose that the preferences of the players are *congruent* with each other in the sense that they agree on the ex-post optimal actions in every state $\omega \in \Omega$:

$$a^* \in \arg\max_{a \in A} u(a, \omega) \Longleftrightarrow a^* \in \arg\max_{a \in A} v(a, \omega). \tag{11}$$

When (11) holds, it is clear that the material payoff of the sender is maximized when she provides full information to the receiver. Hence, we have $U^* = \bar{U}$, and Theorem 2 applies.

An obvious setting with congruent preferences is when players' material interests are *perfectly aligned.* Namely, when there exists a strictly increasing function $\Psi : \mathbb{R} \to \mathbb{R}$, such that $u(a, \omega) = \Psi(v(a, \omega))$ for all $(a, \omega) \in A \times \Omega$. Panel (a) in Figure 2 depicts the set of implementable material payoff profiles for the case where $\Psi(\cdot)$ is a linear function. In this case, the mapping between the *expected payoffs* of the two players is also a linear one. Consequently, the D1 equilibria are not only payoff-equivalent to the sender (as already asserted by Theorem 1), but also to the receiver. A particularly simple equilibrium is one in which the sender always commits to an information structure that either reveals everything (i.e., recommending an ex-post optimal action) or reveals nothing (i.e., recommending an ex-ante optimal action) to the receiver, with the frequency of the former action decreasing in the sender's type. Restricting to this class of equilibrium information structures, a more image-concerned sender (captured by either a higher θ or ϕ) will transmit less information to the receiver, therefore leading to a lower receiver welfare.

Perfect alignment of material interests is by far not the only setting that implicates congruent preferences. In Appendix A.8.1, we provide an example where the players disagree on the second-best actions, even though they concur on the first-best actions in every state. Hence, although the material interests of the players are not perfectly aligned, the congruency

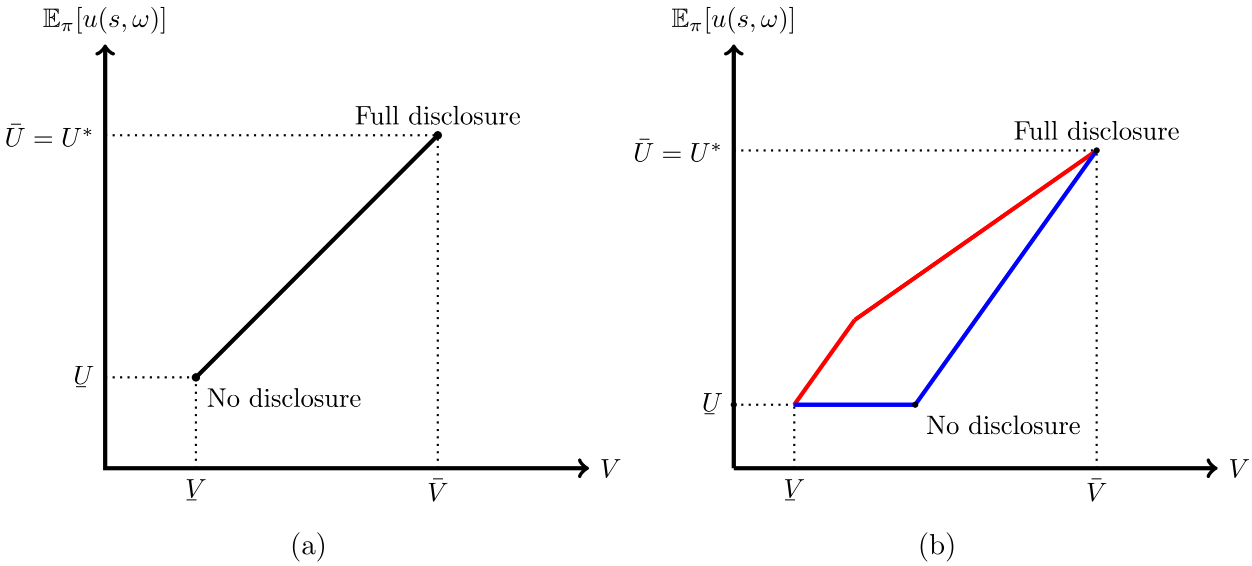


Figure 2: The equilibrium set of implementable payoffs in settings with $U^* = \bar{U}$. Panel (a) represents a game where the preferences of the players are perfectly aligned, with $u(a,\omega) = v(a,\omega)$, $\bar{V} = 0.5$ and $\underline{V} = 0.1$. Panel (b) represents a game the players' preferences are not perfectly aligned, but the congruency condition (11) holds (see Appendix A.8.1 for details). The upper curve (colored in red) in the graph depicts the utility-frontier of the Pareto-optimal D1 equilibria, while the lower curve (colored in blue) corresponds to the utility-frontier of the Pareto-worst D1 equilibria.

condition (11) is satisfied. Panel (b) in Figure 2 visualizes the set of implementable material payoff profiles in this example. Especially, the upper curve in red (the lower curve in blue) delineates, for any given level of the sender's payoff $V \in [\underline{V}, \bar{V}]$, the maximum (minimum) payoff that the receiver can attain. Consequently, in any Pareto-extremal equilibrium, different sender types will "line up" along these curves to forgo their material utilities, giving rise to the patterns of monotonicity/quasi-convexity highlighted by Theorem 2.

Example 2: Quadratic loss. Suppose that $A = \Omega = [0,1]$. The receiver's utility function is $u(a,\omega) = -(a-\omega)^2$. The sender garners a material payoff $-(a - a^*(\omega,\theta))^2$ for every $(a,\omega) \in A \times \Omega$. The sender's bliss point may vary by her type: $a^*(\omega,\theta) = f(\theta)\cdot\omega + g(\theta)$. We will illustrate how this potential type-dependence of the sender's payoff from material outcomes can be accommodated within our framework.

Communication games in which players' preferences take the form of such quadratic loss functions were popularized by the seminal work of Crawford and Sobel (1982), and they have received considerable attention in the information design literature (see, e.g., Galperti, 2019; Jehiel, 2015; Kamenica and Gentzkow, 2011; Smolin and Yamashita, 2022; Tamura, 2018). In the classic information design setting devoid of image concerns, the players' incentives are purely governed by their disagreement over the optimal action plan: while the receiver wants to exactly match the state ($a = \omega$), the sender may have a systematically different target ($a = a^*(\omega, \theta)$). The current example, as well as Example 5 in the next subsection, examine the conditions under which introducing image concerns would mitigate or amplify the above misalignment of preferences and consequently lead to more or less information transmitted in equilibrium.

In Appendix A.8.2, we show that if $f(\theta) > 0.5\ \forall \theta \in [0, 1]$ is satisfied, then the initial quadratic-loss game is equivalent to one in which the sender has the material payoff function $v(a, \omega) = u(a, \omega)$ and the image payoff function $\hat{w}(p(\eta), \theta) = w(p(\eta), \theta)/(2f(\theta) - 1)$. Here, $w(\cdot)$ denotes the image payoff function of the initial game. This transformation manifests that the players' interests are sufficiently aligned under the current specification, insomuch that a sender purely guided by material interests would be willing to share all information with the receiver. However, if function $\hat{w}(\cdot)$ satisfies the key condition (1) – which can be the case, for instance, if $f'(\cdot) < 0$, meaning that higher types put less weight on the state-dependent term relative to the state-independent target $g(\cdot)$ – then both Theorems 1 and 2 apply. They jointly imply that all types (except possibly type 0) will withhold information from the receiver for signaling purposes. Moreover, given that $v(a, \omega) = u(a, \omega)$, the equilibrium payoffs of both the sender and the receiver are uniquely pinned down by the D1 criterion.

Theorem 2 and the examples following it are related to the literature on "bad reputation" in repeated games (see, e.g., Ely *et al.*, 2008; Ely and Välimäki, 2003). An overarching finding of this literature is that reputational concerns harm a long-lived player who repeatedly interacts with short-lived players if they are based on a desire to separate from a bad type rather than to mimic a good commitment type (see the discussion in Mailath and Samuelson, 2006). The forces behind our results are quite different: the sender tries to separate herself from the type that is *least image-concerned*, which requires her to avoid taking the strategy that would be *endogenously* chosen by the latter. In the current set-up, that strategy happens to be the one that maximizes the material payoffs of both players. In other circumstances, such a desire for separation of the sender could also benefit the receiver, or its effects might depend on equilibrium selection. We will further explore these intricacies in the sections that follow.

3.2.2 When will sender's image concerns be unequivocally beneficial?

When does the presence of image concerns benefit the receiver, irrespective of which D1 equilibrium is selected? Analogous to the previous subsection, we focus on settings in which the following simple sufficient condition holds: a sender who acts out of pure material interest will implement the *no-information payoff* for the receiver. Theorem 3 below summarizes some key properties of the equilibrium set in such settings.

Theorem 3. *If $U^* = \underline{U}$, the receiver can never be harmed by the presence of the sender's image concerns. Moreover,*

(i) provided that $\phi < \bar{\phi}$ (so that the cutoff tpe $\hat{\theta} > 0$), there exists a D1 equilibrium in which the receiver is strictly better off compared to the case without image concerns;[14]

(ii) in any Pareto-optimal D1 equilibrium, the receiver's expected payoff is quasi-concave with respect to the sender's type;

(iii) in any Pareto-worst D1 equilibrium, the receiver's expected payoff is increasing with respect to the sender's type.

Both the proof and the intuition of Theorem 3 are analogous to Theorem 2, and therefore omitted to avoid repetition. Below, we illustrate the main insights of the theorem through several examples.

Example 3: No gain from persuasion. Kamenica and Gentzkow (2011) characterize when a sender purely driven by material interests can benefit from persuasion. That is when she can do *strictly* better than providing no information (or always recommending an ex-ante optimal action) to the receiver. When this is *not* the case, $U^* = \underline{U}$ obviously holds, so Theorem 3 applies.

A concrete setting where the sender would not want to share any information in the absence of image concerns is when players engage in a zero-sum (or constant-sum) game. Namely, when there exists a constant $K \in \mathbb{R}$, such that $v(a, \omega) + u(a, \omega) = K$ for all $(a, \omega) \in A \times \Omega$. Panel (a) in Figure 3 depicts the set of implementable material payoff profiles in such a game. As with perfectly aligned interests (see Example 1), the linear mapping between the

[14] This statement remains valid when $\phi \geq \bar{\phi}$ under the additional assumption that an information structure $\pi \in \Pi^*$ satisfying $\mathbb{E}_\pi[v(s, \omega)] = \underline{V}$ and $\mathbb{E}_\pi[u(s, \omega)] > \underline{U}$ exists.

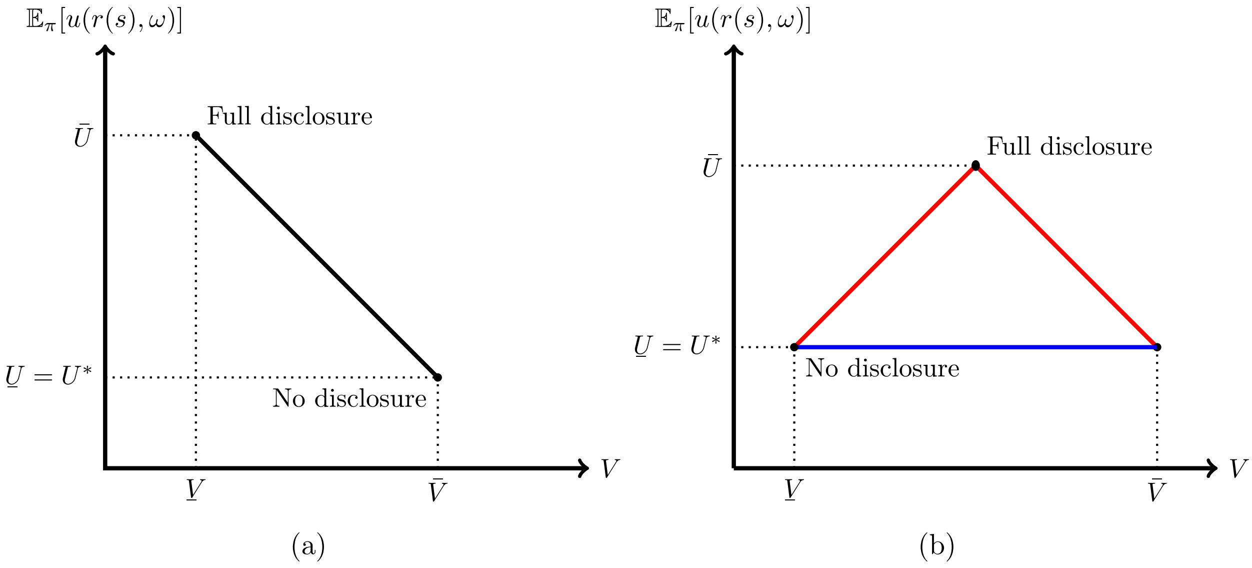


Figure 3: The equilibrium set of implementable payoffs in settings with $U^* = \underline{U}$. Panel (a) represents a game where the players have exactly opposite interests over the material outcomes, with $u(a,\omega) = -v(a,\omega)$, $\bar{V} = 0.5$ and $\underline{V} = 0.1$. In panel (b), we have a game with partially conflicting interests as described in Example 5. The upper curve (coloured in red) in the graph depicts the utility-frontier of the Pareto-optimal D1 equilibria, while the lower curve (coloured in blue) corresponds to utility frontier of the Pareto-worst D1 equilibria.

players' expected payoffs implies that all D1 equilibria are payoff-equivalent to both players. A particularly simple equilibrium is one in which the sender always commits to an information structure that reveals either everything or nothing about the true state, with the frequency of the former action increasing in the sender's type. In this case, a more image-concerned sender (captured by higher θ or ϕ) will transmit more information to the receiver, therefore increasing his welfare.

Example 4: Quadratic loss (continued). Consider again the quadratic-loss games introduced in the previous subsection. In Appendix A.8.2, we show that under the condition $f(\theta) < 0.5$ $\forall\theta \in [0,1]$, the original game is strategically equivalent to one where the sender has a material function $v(a,\omega) = (a-\omega)^2$ and an image payoff function $\hat{w}(p(\eta),\theta) = w(p(\eta),\theta)/(1-2f(\theta))$. Since $v(a,\omega) + u(a,\omega) = 0$ $\forall(a,\omega) \in A \times \Omega$, the transformed game is a zero-sum one regarding the players' material payoffs. Provided that condition (1) holds for $\hat{w}(\cdot)$ – which can occur, for instance, if the interests of higher types are more aligned with the receiver in the sense that $f'(\cdot) > 0$ – both Theorems 1 and 3 apply. Thus, the presence of image concerns will trigger all sender types (except possibly type 0) to share information with the receiver, which they would be reluctant to do in a pure persuasion setting.

Next, we introduce two widely-studied examples in which the sender would partially disclose the state if only the persuasion motive is present, yet the receiver's payoff remains minimal. This demonstrates that the applicability of Theorem 3 is *not* limited to settings in which the sender would not share any information in the absence of image concerns. Intuitively, the optimality of partial disclosure may be compatible with the premise $U^* = \underline{U}$ of Theorem 3, because having access to partial information does not guarantee that the receiver can do *strictly* better *on average* than taking his prior-optimal action. This observation is important and may prove useful beyond the examples below because it is known that partial disclosure is optimal in many pure persuasion settings. For instance, Jehiel (2015) shows that this is typically the case when the information of the sender is higher dimensional than the action space of the receiver; Kolotilin and Wolitzky (2020) and Kolotilin, Corrao and Wolitzky (2022a) provide other sufficient conditions in a setting that allows utilities of the sender and receiver to be non-linear in the state.[15]

Example 5: State-independent sender preferences, I. Suppose that $A = \Omega = \{0, 1\}$, and the players' material payoff functions are $v(a, \omega) = a$ and $u(a, \omega) = \mathbb{1}_{a=\omega}$. Thus, while the receiver wants to match the state, the sender's preference over material outcomes is state-independent: she always prefers the receiver to take the high action. This persuasion setting is most vividly embodied by the prosecutor-judge example in Kamenica and Gentzkow (2011). Since the state space is binary, we use μ_0 to denote the prior likelihood of the state being $\omega = 1$. We assume $\mu_0 \in (0, 0.5)$ so that $a = 0$ is the receiver's optimal action given the prior. Clearly, releasing no information minimizes the sender's material payoff. At the same time, Kamenica and Gentzkow (2011) show that partial information disclosure is optimal for the sender when she has no image concerns. Nevertheless, under the optimal disclosure policy, the receiver weakly prefers his prior-optimal action regardless of the signal realization, so her expected payoff is the same as under no information (i.e., $U^* = \underline{U}$). Hence, all results of Theorem 1 and Theorem 3 apply.

We present two simple classes of information structures that one may use to describe the Pareto-optimal and the Pareto-worst D1 equilibria in closed form, respectively. For every $q \in [0, 2\mu_0]$, define an information structure $\overline{\pi}^q$ as follows: Conditional on the true state, the

[15] See, e.g., Theorem 2 in Kolotilin and Wolitzky (2020). Optimal partial disclosure has been shown to take the form of censorship (Kolotilin, Mylovanov and Zapechelnyuk, 2022b), nested intervals (Guo and Shmaya, 2019), (p)-pairwise signals (Kolotilin and Wolitzky, 2020; Terstiege and Wasser, 2022), or conjugate disclosure (Nikandrova and Pancs, 2017).

signal $s = 1$ is drawn with probability

$$\bar{\rho}(\omega; q) = \begin{cases} \min\left\{\frac{q}{\mu_0}, 1\right\} & \text{if } \omega = 1, \\ \max\left\{\frac{q-\mu_0}{1-\mu_0}, 0\right\} & \text{if } \omega = 0. \end{cases} \tag{12}$$

With the remaining probability $1 - \bar{\rho}(\omega; q)$, the signal $s = 0$ is sent to the receiver. One can check that $\bar{\pi}^q$ satisfies the obedience constraint (i.e. $\bar{\pi}^q \in \Pi^*$), and that it induces the receiver to choose the action $a = 1$ exactly with probability q. While there can be other information structures that induce the same marginal distribution of actions, all of them will be Pareto-dominated by $\bar{\pi}^q$ (see Appendix A.8.3 for a formal proof). For instance, consider the information structure $\underline{\pi}^q$ defined as follows: Conditional on the true state, the signal $s = 1$ is drawn with probability

$$\underline{\rho}(\omega; q) = \begin{cases} \frac{q}{2\mu_0} & \text{if } \omega = 1, \\ \frac{q}{2(1-\mu_0)} & \text{if } \omega = 0. \end{cases} \tag{13}$$

With the remaining probability $1 - \underline{\rho}(\omega; q)$, the signal $s = 0$ is sent to the receiver. With this information structure, the sender can also nudge the receiver to choose the high action with probability q. However, the probability that the receiver takes the "right" action is just $1 - \mu_0$ under $\underline{\pi}^q$ for any $q \in [0, 1]$, which he could also achieve by simply sticking to his prior-optimal action $a = 0$. This is clearly the worst possible outcome for the receiver, so he would prefer $\bar{\pi}^q$ over $\underline{\pi}^q$. All things considered, there must exist a Pareto-optimal (Pareto-worst) equilibrium in which each sender type θ uses the information structure $\bar{\pi}^{q(\theta)}$ ($\underline{\pi}^{q(\theta)}$), and in which $q(\theta)$, the total probability that the receiver would take the action $a = 1$, is decreasing in the sender's type. Panel (b) in Figure 3 depicts the receiver welfare in both equilibria, delineating the whole set of implementable payoff profiles for the receiver.

A salient feature of the Pareto-optimal equilibrium is that the receiver's welfare can be non-monotone in the sender's type. This non-monotonicity arises as follows: types towards the lower end of the separating interval strive to enhance their reputation by releasing more information about the state. However, the additional information needed for achieving separation through this way may be substantial, to the extent that an intermediate type is already compelled to provide complete information. Then, even higher types within the separating interval can only credibly signal their type by sacrificing further material utility in ways that also harm the receiver. By contrast, in the Pareto-worst equilibrium, all sender types minimize the receiver's payoff to his reservation utility $\underline{U}$.

Example 6: State-independent sender preferences, II. Let $A = \{0, 1\}$, $\Omega = [0, 1]$, $v(a, \omega) = a$ and $u(a, \omega) = a \cdot \omega + (1 - a) \cdot \underline{u}$, where $\underline{u} \in (0, 1)$ can be interpreted as the value of the receiver's outside option ($a = 0$). We assume that $\underline{u} > \mathbb{E}_{\mu_0}[\omega]$. Thus, the receiver's default action is $a = 0$, and $\underline{u}$ will also be his expected payoff under no information. Further, in the absence of image concerns, the optimal strategy of the sender would extract all the surplus from the receiver (see Section V. B in Kamenica and Gentzkow (2011)). Taken together, we have $U^* = \underline{U} = \underline{u}$, so both Theorems 1 and 3 can be applied to study this example.[16]

3.2.3 When will the welfare implications be ambiguous?

In general, the receiver's payoff may be strictly between his full- and no-information payoffs in the canonical setting without image concerns. Our last formal result confirms that in this case, whether the sender's image concerns will be beneficial or detrimental for the receiver can be uncertain in the sense that the direction of the effect depends on the selected equilibrium.

Theorem 4. *If $U^* \in (\underline{U}, \bar{U})$, whether the receiver benefits from or is harmed by the presence of the sender's image concerns can depend on the selected equilibrium and the type distribution. In particular, provided that ϕ is small enough, there will always be two co-existing equilibria: (i) a D1 equilibrium in which the receiver is strictly better off and Blackwell-more information is transmitted and (ii) a D1 equilibrium in which the receiver is and strictly worse-off and Blackwell-less information is transmitted, both relative to the setting without image concerns.*[17]

As we alluded before, the ambiguous effect of image concerns is largely due to that standard refinements, including the D1 criterion, do not fully pin down the structure of the

[16] Under the additional assumption that ω is uniformly distributed, we can construct two simple classes of information structures to describe the Pareto-extremal equilibria in closed form. In particular, the Pareto-optimal information structures "censor" the states below a threshold that varies with θ. By contrast, the Pareto-worst information structures "censor" the states within some intermediate interval. The details of the construction are provided in Appendix A.8.4.

[17] In the other extreme, when the image concern parameter ϕ is sufficiently large, all equilibria become fully pooling. In this scenario, the receiver fares better (worse) than in the setting without image concern if all sender types use an information structure that implements the minimal material payoff $\underline{V}$ while giving the receiver $U > U^*$ ($U < U^*$). Further, note that the proof of part (iii) of Theorems 2 is not dependent on the condition $U^* = \bar{U}$. Hence, the quasi-convexity property of the Pareto-worst equilibrium continues to hold even when $U^* < \bar{U}$. Likewise, the quasi-concavity property of the Pareto-optimal equilibrium, as identified by Theorem 3, remains valid here despite the condition $U^* > \underline{U}$.

sender's equilibrium strategy, although they necessitate that the sender's interim payoffs are equivalent across all equilibria. The vital obstacle is that standard refinements rule out equilibria by discerning unreasonable payoff incentives of the sender, e.g., the D1 criterion rejects equilibria with off-path beliefs that put mass on types who gain less from deviation. However, the abundance of possible information structures allows diverse choices that lead to the same payoff for the sender. Thus, these choices of information structures cannot be further differentiated by standard refinements, notwithstanding the possibility of having vastly different implications for the receiver's welfare.

Remark on optimal information structures. We view the multiplicity of equilibrium information structures in our model as a qualification of the information design approach, rather than a drawback. Following Schelling (1980), one may interpret the multiplicity as a manifestation of different cultures of communication. As Myerson (2009) emphasizes, selecting among multiple equilibria is a "fundamental social problem", and recognizing this problem "can help us to better understand the economic impact of culture". Applying Schelling's approach to information design, external factors and details of a specific application can be used to qualify a class of information structures and thereby select an equilibrium.

On a related note, our reduced-form characterization of equilibria adds to the recent discussion regarding a common critique of the information design approach. The design approach distinguishes itself from other theories of sender-receiver games by allowing the sender to choose (and commit to) *any* information structure. The critique, as summarized by Kamenica, Kim and Zapechelnyuk (2021), argues that "optimal information structures can be infeasible or difficult to implement in practice". A strand of the literature has addressed this issue by identifying sufficient conditions for simple information structures to be optimal among all information structures (e.g., Ivanov, 2021; Kolotilin *et al.*, 2022b; Kolotilin and Wolitzky, 2020). Our analysis shows, in a setting extended from the canonical one without image concerns, that a class of simple information structures (e.g. the censoring of available information) is consistent with equilibrium requirements under the condition that it can fully implement all possible material payoffs of the sender. Thus, this condition can serve as a formal justification for focusing on some specific class of information structures in applications.

4 Applications

4.1 Self-Signaling and Willful Ignorance

Since the sender and the receiver can be interpreted as two selves of the same agent, our model applies to situations of self-signaling (Bodner and Prelec, 2003). In a typical self-signaling situation, an individual forms beliefs about her own abilities (Köszegi, 2006; Schwardmann and Van der Weele, 2019), moralities (Bénabou and Tirole, 2006; Chen and Heese, 2023; Grossman and Van der Weele, 2017) or other inner characteristics such as self-control (Bénabou and Tirole, 2002, 2004) based on her past conduct, from which she may also derive a direct flow of utility.

In the context of self-signaling, our model is similar to, e.g., Bénabou and Tirole (2002) and Grossman and Van der Weele (2017), in that the signaling is via the sender-self's information choice. The main difference is that we do not restrict the sender-self's choice to a prespecified class of information structures. The assumption that the sender can fully commit to any information structure, which plays a central role in the Bayesian persuasion literature and is often considered somewhat extreme, can be quite natural in the dual-self setting. It simply captures the intra-personal transparency of information acquisition, that is, the sender-self cannot distort information or knowingly lie to the receiver-self. This point is particularly evident with a binary state, because it has been shown that in such settings Bayesian persuasion is equivalent to a dynamic information acquisition game where all selves of an agent observe public information arriving according to a drift-diffusion process (e.g. Chen and Heese, 2023; Henry and Ottaviani, 2019; Morris and Strack, 2019).[18]

To showcase the applicability of our model in self-signaling situations, consider an agent who is faced with a mental task. Both selves of the agent receive the same state-dependent material payoff $v(a, \omega)$ from an action choice. Ultimately, the receiver-self of the agent will decide which action a to take. Nevertheless, the sender-self can "cheat" by acquiring some information about the state. Formally, she can choose a joint distribution of the state and signal, and then use it to generate an action recommendation to the receiver-self. The sender-self also possesses private information regarding the agent's capability to solve the task without informational assistance, which is encapsulated in the agent's type θ. In particular, the sender-self

[18] The drift-diffusion model is a well-established model of information processing in neuroeconomics and psychology. See, e.g., Fehr and Rangel (2011); Fudenberg, Newey, Strack and Strzalecki (2020); Krajbich, Oud and Fehr (2014); Ratcliff, Smith, Brown and McKoon (2016) and the references therein.

knows that, with probability $f(\theta)$, the receiver-self will be able to directly observe the true state in the action-taking stage, regardless of which information structure the sender-self has chosen. The function $f(\cdot)$ is strictly increasing, reflecting the idea that higher types are associated with higher abilities. The agent further derives a "diagnostic utility" $\psi \cdot \mathbb{E}_\eta[\tilde{\theta}]$ from being perceived as a high type by her receiver-self, where $\psi > 0$. It is straightforward to verify that this dual-self game maps into the following specification of our general model: the sender has the utility function $v(a,\omega) + (\delta f(\theta) + \psi \cdot \mathbb{E}_\eta[\tilde{\theta}])/(1 - f(\theta))$, where $\delta \equiv \mathbb{E}_{\mu_0}[\max_{a\in A} v(a,\omega)]$ is a constant, while the receiver has the utility function $u(a,\omega) = v(a,\omega)$.[19]

Our previous results for such common-value settings (see Example 1 in Section 3.2.1) are quite clear-cut. In equilibrium, higher types will "self-handicap" by acquiring less accurate information, for the goal of boosting their egos. Such handicapping behavior, which was similarly found in Bénabou and Tirole (2002), unambiguously reduces the material payoff of the agent. This result contributes to the growing body of research on information avoidance, which studies the widely-documented phenomenon that decision makers may willfully abstain from obtaining free and useful information for, e.g., psychological or cognitive reasons. For an excellent survey on this topic, see Golman *et al.* (2017).[20]

4.2 On Transparency in Organizations

We revisit the question of transparency in organizations, as studied by Jehiel (2015). More specifically, the question is when a manager (sender) of an organization prefers being opaque about what she knows in a moral hazard interaction with a worker (receiver). In what follows, we identify a new force that drives intransparency in organizations, which rests on reputational concerns of the manager.[21]

Reputational concerns in organizations might arise internally from the norms or guidelines of a company, as well as the explicit or implicit incentives of employees to signal compliance. To make this point concrete, we follow Jehiel (2015)'s motivating example and formulate

[19] Each sender type θ chooses $\pi \in \Pi^*$ to maximize $(1-f(\theta))\cdot\mathbb{E}_\pi[v(s,\omega)]+\mathbb{E}_{\mu_0}[\max_{a\in A} v(a,\omega)]\cdot f(\theta)+\psi\cdot\mathbb{E}_\eta[\tilde{\theta}]$, which is equivalent to maximizing $\mathbb{E}_\pi[v(s,\omega)]+(\delta f(\theta)+\psi\cdot\mathbb{E}_\eta[\tilde{\theta}])/(1-f(\theta))$. Assuming that $f(\cdot)$ is continuously differentiable, the function $w(p,\theta) = (\delta f(\theta) + \psi p)/(1 - f(\theta))$ is continuously differentiable in both of its arguments and satisfies our condition (1).

[20] Our result is also related to a strand of literature in social psychology, which documents that individuals exhibit a wide array of behavior that is factually bad for them but presumably useful for self-presentation; see, e.g, Crocker and Park (2004); Schlenker (2012).

[21] Jehiel (2015) focuses on two distinct forces that make full transparency suboptimal, which concern either the sensitivity or the concavity of agents' utilities over actions in different states.

the moral hazard interaction through a preference setting with $A = \Omega = [0, 1]$ and quadratic losses à la Crawford and Sobel (1982). The worker's utility function is $u(a, \omega) = -(a-\omega)^2$, so his effort bliss point equals exactly to the state ($a = \omega$). However, from the viewpoint of the company's senior management, the effort bliss point is $\beta \cdot \omega$, where $\beta > 1$. Thus, the ideal level of effort is consistently higher for the senior management than for the worker. As for the (mid-level) manager, she derives a material payoff $-(a - f(\theta) \cdot \omega)^2$ from the worker's effort, where $f(\theta) \equiv (\beta - 1) \cdot \theta + 1$ ensures that the manager's preferences over effort always extrapolate between those of her boss and her subordinate. Furthermore, the strictly increasing property of $f(\cdot)$ signifies that higher types have internalized the senior management's point of view more strongly. Last, the manager likes to be perceived as a high type, or in other words, as being "compliant" to the preferences of the higher-ups. Formally, the manager receives an image payoff $\phi \cdot \theta \cdot \mathbb{E}_\eta[\tilde{\theta}]$, where η is interpreted as the senior management's belief about the manager's type. Overall, our payoff specification posits that higher types care more about the impression that they leave on the boss. This seems reasonable because, presumably, these are the types that are more committed to a career in the current company.

What do the incentives of signaling compliance to the higher-ups imply in terms of transparency and organizational performance? Similar to Jehiel (2015), we have a fully transparent benchmark in the current quadratic-loss setting: If the manager has no reputational concerns ($\phi = 0$), then all manager types would fully communicate all information about the state to the worker.[22] However, Theorems 1 and 2 jointly imply that, when the manager worries about the (explicit or implicit) review of her compliance by the senior management, all types except possibly type 0 will adopt strategies that conceal information from the worker. Thus, the motive of "pleasing the boss" can be a compelling source of intransparency in organizations. This lack of transparency, in turn, harms organizational performance, because in expectation the worker's effort choice will be further away from the company's bliss point, i.e. the senior management's, compared to the fully transparent case.

It is perhaps unrealistic to think that the desire to establish a reputation among one's colleagues would always have an unequivocally negative effect on the transparency of the organization. For instance, instead of signaling compliance to the higher-ups, in some workplaces managers may want to signal altruism to their subordinates (Ellingsen and Johannesson, 2008). In those settings, it might seem natural to expect that the concern for reputation

[22] See Section V.A of Kamenica and Gentzkow (2011). For details specific to our setting, see also Appendix A.8.2 and the analysis of Example 2 in Section 3.2.1.

would encourage the manager to share more information with the workers, therefore enhancing the transparency of the organization. The caveat here is that one must consider what the manager would have done in the absence of such reputational concerns. It is possible that, with pure persuasion motives, the manager would disclose partial information about the state to the worker. Then, according to Theorem 4, whether the manager's reputational concerns will drive a more or less transparent organization may hinge on equilibrium selection, which, in turn, can be determined by factors such as social norms and/or the corporate culture of the organization. Such informal factors of organizations are surveyed and discussed in, e.g. Hermalin (2001) and Kreps (1990).

Taken together, the application in this section provides insights into a recent debate on the downsides of hierarchical structures in organizations. Specifically, there are concerns that since attention will naturally be directed up the hierarchy, performance in traditional hierarchical organizations may suffer from the managers focusing too much on "pleasing their bosses" rather than "helping their teams" (Dillon, 2017). To this end, our application provides a game-theoretic model in which pleasing-one's-boss schemes arise and are shown to harm the organization. Our model also offers a novel rationale for why many (but certainly not all) companies nowadays rely on committees to conduct performance evaluations instead of delegating these decisions solely to direct superiors.[23] Intuitively, such arrangements should alleviate the managers' signaling considerations when making pivotal decisions, which, according to our theory, can potentially enhance transparency and improve the performance of the organization.

4.3 Populist Sentiments and Policy Stagnation

In a seminal study, Fernandez and Rodrik (1991) address why governments often fail to adopt reforms considered efficiency-enhancing by experts, which they describe as "one of the fundamental questions of political economy". Indeed, this question is particularly puzzling in the current era, where political leaders emphasize the importance of science and evidence-based policy making for progress and growth.[24] Fernandez and Rodrik (1991) demonstrate that such policy stagnation may occur when voters are uncertain about the idiosyncratic

[23] In 2011, the Society for Human Resource Management surveyed 510 organizations with 2,500 or more employees and found that a majority (54%) of these organizations use formal committees as part of their performance evaluation process.

[24] See, e.g., Mallapaty (2022), Prillaman (2022), and the article "Politics will be poorer without Angela Merkel's scientific approach" by the editorial board of *Nature* (2021) .

impacts of the reform ex ante, therefore rejecting it even though the reform for sure will benefit a *majority* of the democratic public ex post and is welfare-enhancing overall. However, it remains unclear why such uncertainty persists, as the government could in principle seek to educate voters about the potential consequences of the reform, especially given the increasing availability of data and development of information technology. In what follows, we use our framework to show that this phenomenon can be rationalized by the (over-)disciplining effect of policymakers' interest in cultivating a favorable public image.

To this end, we describe a stylized model of politics in which information first flows from experts to a politician, and further to a democratic public (represented as a group of voters) who then accept or reject a reform accordingly. The public's prior opinion is marked by reform skepticism. That is, absent persuasive new information about the potential consequences, the public would reject the reform outright, leading to policy stagnation as in Fernandez and Rodrik (1991). The politician could ask experts to provide such information to the public in principle. At the same time, the politician has reputational concerns: the public may perceive her advocacy for the reform as driven by personal interests, hurting her chances in future elections.[25] One may expect these reputation concerns to encourage information sharing, as the politician tries to appear neutral. Yet paradoxically, we demonstrate that these very concerns can perpetuate an equilibrium in which the public remains uninformed. In particular, the public may remain anchored in their initial skepticism towards the reform even when implementing it would actually benefits *all* voters and when the politician could have committed to share that information.[26]

The formal model is as follows. In the first stage, the politician can acquire information about a binary state $\omega \in \Omega = \{0, 1\}$, which is payoff-relevant for a proposed reform being publicly debated. To obtain this information, the politician can commission a study π that specifies a distribution of results for each possible state. For instance, the politician may appoint an unbiased expert who truly knows the subject to lead the study, which would allow her to always uncover the true state. Alternatively, the politician could select an expert who is known to be biased, e.g. towards the reform, to investigate the matter, in which case a result supporting the reform probably would be less informative relative to one opposing it.

[25] The effect of election incentives on politicians' conduct is a prominent focus in the literature on electoral accountability, as surveyed by Ashworth (2012) and Duggan and Martinelli (2017).

[26] In a similar vein, Kartik and McAfee (2007) study electoral competition where a candidate's policy choice serves as a signal of her character. In their setting, signaling produces, for example, a failure of the median voter theorem.

In the second stage, the politician observes the study's outcome and then decides whether to keep it private or disclose it to the public in the form of a verifiable report. Implementing the reform ($a = 1$) results in a welfare gain of 1 for every voter when $\omega = 1$ but leads to a welfare reduction of 1 when $\omega = 0$, relative to maintaining the status quo ($a = 0$). The public is initially skeptical of the reform, with a common prior belief leaning toward $\omega = 0$ as the more likely state. Thus, only when the disclosed result is sufficiently compelling to overcome these predispositions, the politician's communication is effective in generating support for the reform. The politician receives a state-independent payoff $w_1(\theta) > 0$ if the reform is adopted, where $w_1(\cdot)$ is strictly decreasing in her private type $\theta \in [0, 1]$; otherwise, her payoff at this stage is zero. The interpretation is that θ is linked to the "corruptness" of the politician, evidenced by her private gains from pushing actions against the interest of the public (i.e. advocating $a = 1$ when $\omega = 0$). Higher values of θ reflect greater alignment with public interest, consistent with lower private benefits from the adoption of the reform.

The third and last stage introduces reputational concerns of the politician. In this stage, the politician runs against another candidate in an election. Each voter receives a payoff $\alpha\theta + \epsilon$ if a politician of type θ is elected to office, where $\alpha > 0$ is a parameter and $\epsilon \in \mathbb{R}$ is a common preference shock (e.g. changes in the economic environment) drawn according to an absolutely continuous cumulative distribution function G. This specification could be interpreted as capturing the intuition that more corrupt politicians are more likely to act against the public's interest once elected. Alternatively, it may simply reflect that voters intrinsically desire more "ethical" politicians in office. We fix the voters' expected payoff from electing the opposing candidate as $\underline{u}$, which satisfies $G(\underline{u}) \in (0, 1)$. Voters observe the preference shock ϵ, but not the politician's type θ. However, they make a Bayesian inference about θ based on the politician's "past record", which in our model amounts to whether and how she previously attempted to influence public opinion about the reform. Thus, voters will support the politician if and only if $\alpha p + \epsilon \geq \underline{u}$, where p is the voters' posterior expectation regarding the politician's type. Accordingly, the likelihood of the politician winning the election is $1 - G(\underline{u} - \alpha p)$. Finally, the politician's payoffs upon winning and losing the election are given by $w_2(\theta)$ and $w_3(\theta)$, respectively. We assume that the ratio $(w_2(\theta) - w_3(\theta))/w_1(\theta)$ is continuously differentiable and strictly increasing in θ, so that less corrupt types care more about the election relative to the reform.[27] This monotonicity assumption ensures that

[27] Given that $w_1(\cdot)$ is strictly decreasing, the desired monotonicity holds if all types are purely office-motivated (i.e., $w_2(\cdot) - w_3(\cdot)$ is a constant function), a setting commonly studied in the literature on electoral

the relevant single-crossing property holds, paving the way for election-driven reputational concerns to shape the politician's communication with the voters.[28]

In Appendix A.10.2, we demonstrate in detail how this political economy model can be solved in reduced form with the previous analysis. Specifically, we show that the equilibrium incentives of the politician can be mapped into a specialized setting of Example 5 studied in Section 3.2. Drawing upon the analysis of that example, we conclude that higher types necessarily commission studies less favorable to the reform, expressing fewer and/or weaker endorsements of the reform to the public. This equilibrium outcome and its key driving force can be intuitively understood in terms of "populist sentiments": the politician seeks to position herself in the debate surrounding the reform – through strategically commissioning and revealing study results – in a way that appeals to the public she is "on their side" (i.e., not corrupt). Indeed, this interpretation aligns with the standard political science definition of populism as "a political philosophy supporting [...] the people in their struggle against the privileged elite" (see the corresponding item in the American heritage dictionary).[29]

A critical insight from our analysis is that the effect of populism on the politician's communication strategy is ambivalent, and does not necessarily translate monotonically into the welfare of the public. To be concrete, consider an increase in α, which could represent heightened populist sentiment among the public, whereby voters become more concerned about the politician's corruption when deciding their electoral support. As we formally show in Appendix A.10.3, this parallels the effect of increasing ϕ – the relative weight that the sender places on image versus material payoffs – in the general model. Intuitively, a larger α strengthens the signaling incentives of the politician because her public image becomes more decisive for the election outcome. Thus, similar to the results from Section 3.2, the public's welfare is non-monotone in α. In particular, when α is sufficiently small, all equilibria are fully separating – meaning voters can fully distinguish between corrupt and non-corrupt politicians – and the payoff from reform choices increase in α in the Pareto optimal equilibrium (see the discussion surrounding Example 5).

However, when α is large, the electoral outcome becomes highly dependent on the per-

competition (Persson and Tabellini, 2002). Moreover, as we formally show in Appendix A.10.1, this monotonicity assumption can also be derived from a setting where the politician is both office- and policy-motivated.

[28] In comparison, if $((w_2(\cdot) - w_3(\cdot))/w_1(\cdot))' < 0$ holds, then more corrupt types have higher incentives to be elected. In this case, the disciplining effect of the election concerns disappears – the unique equilibrium is such that all types promote the reform in the same way as in the benchmark game without those concerns.

[29] See also Mudde (2004), Acemoglu, Egorov and Sonin (2013), and the recent VoxEU debate on populism (available at https://cepr.org/debates/populism).

ceived morality of the politician. Recognizing the substantial electoral ramifications of a tarnished reputation, the politician may exhibit reluctance to endorse the reform even when it is the right course of action. In the most extreme case, all politician types conform to the public's prior skepticism by recommending the status quo with probability one, regardless of the state. As such, the public learns nothing about either the reform or the politician's integrity. Critically, even when the reform would enhance welfare for all parties involved, the public's equilibrium belief remains equal to their skeptical prior, so the status quo persists and policies stagnate. This result underscores that the important observation made by Fernandez and Rodrik (1991) – that uncertainty surrounding policy outcomes can foster "resistance to reform" – remains valid in an extended model where the public's information is endogenous and all uncertainty could, in principle, be resolved.

5 Conclusion

We have developed a novel framework that enables studying the strategic disclosure of information as jointly determined by two counterveiling forces: the standard motive to persuade an audience towards actions preferred by the sender, and the relatively underexplored motive to manage impressions regarding unobserved characteristics of the sender. Our main results delineate the Pareto frontier of the equilibrium set, highlighting that the sender's information choices and the receiver's welfare can exhibit non-monotonicity with respect to the relative strength of the two motives. Since concerns about one's image admit diverse interpretations ranging from psychological preferences to reputational considerations in dynamic interactions, our model offers broad applicability. We demonstrate this versatility across multiple contexts, generating new insights into a number of issues that have received considerable attention from researchers and practitioners. In particular, we leverage the model to discuss the egotistic rationale behind information avoidance, establish the link between harmful intransparencies in organizations and managers' career concerns, and illustrate how heightened populism may contribute to policy stagnation.

We close by suggesting two directions for future research, each of which relaxes some restrictions made in our current framework. First, our model assumes that the sender's payoff is separable with respect to the material allocations and her type-specific gains from reputation. This quasi-linear structure – which is commonly employed in applied studies – greatly simplifies our analysis, as it ensures that the relevant single-crossing property holds at the interim stage. Plainly, the analysis continues to hold if one directly imposes this single-crossing property on the sender's interim payoffs. However, this assumption remains restrictive, because it requires the payoff difference between any pair of *probability distributions* over ex-post allocations to be single-crossing in the sender's type. As discussed by Kartik, Lee and Rappoport (2023) (and see also Kushnir and Liu, 2019), only a limited set of ex-post payoff specifications could generate this property. Nevertheless, Chen, Ishida and Suen (2022) recently provide a general analysis for costly signaling under double-crossing preferences, which nest single-crossing as a special case. By identifying environments where the double-crossing property naturally arises at the interim stage, one could combine our reduced-form approach and the techniques from Chen *et al.* (2022) to obtain additional insights.

Second, our model precludes any correlation between the state and the sender's type, which potentially limits its suitability for certain settings and warrants further investigation. For instance, the alignment of preferences between managers and their supervisors may vary across states, and the necessity of reform could correlate with the corruptness of the incumbent politician. Such correlations naturally lead to an informed principal problem, which is known to be highly challenging in the literature (Myerson, 1983). Although a comprehensive analysis tackling this intricate issue is beyond the scope of our paper, we note that there have been several exciting works recently aiming to develop a toolkit for studying informed principal problems in the context of information design (e.g. Koessler and Skreta, 2023; Zapechelnyuk, 2023). Integrating these cutting-edge approaches with our framework provides fertile ground for new applications.

Appendix

A.1 A Revelation Principle in Our Setting

In this section, we establish a revelation principle in our setting. Also, we explain how the tie-breaking rule assumed in the main text allows for simplifying the notation significantly without changing any results (given a mild assumption on the players' payoffs, which is satisfied by all examples and applications discussed in our paper). Unlike in the main text, we consider a more general environment where the signal space of each information structure is not restricted to be contained in the receiver's action space. Additionally, the receiver is allowed to choose a mixed action or break ties in any manner he prefers contingent on the sender's choice of information structure and the signal realization.

Take any perfect Bayesian equilibrium of the (more general) sender-receiver game. Let σ be the associated strategy of the sender, specifying an information structure $\sigma(\theta) \in \Delta(\Omega \times \mathcal{S})$ for each type $\theta \in \Theta$, where $\mathcal{S}$ is a signal space that may or may not be contained in A. Let $\hat{a}_\pi : \text{supp}(\pi) \to \Delta(A)$ and $\eta(\pi) \in \Delta(\Theta)$ be the signal-contingent decision rule and posterior belief that the receiver adopts in this equilibrium, following each choice of information structure π by the sender, respectively. The equilibrium notion requires that, given the receiver's decision rules $\hat{a}_\pi$ and belief system $\eta(\cdot)$, each $\sigma(\theta)$ is an optimal choice for the corresponding sender type $\theta \in \Theta$. At the same time, given the sender's strategy σ, the decision rules $\hat{a}_\pi$ are sequentially rational for the receiver, and the belief system $\eta(\cdot)$ is consistent with Bayes' rule. In the following, we summarize an equilibrium by $(\sigma(\cdot), a_{(\cdot)}, \eta(\cdot))$.

We proceed in three steps to establish the revelation principle. The first step concerns the on-path behaviour: We show that it is without loss to focus on equilibria in which each sender type chooses an information structure from the set Π^* (recall that Π^* consists of all information structures that have a signal space $\mathcal{S} \subseteq A$ and satisfy the obedience constraints), and in which the receiver consistently obeys the sender's recommended action on the equilibrium path.

Step 1. *For any equilibrium $(\sigma(\cdot), a_{(\cdot)}, \eta(\cdot))$ of the (more general) sender-receiver game, there is an equilibrium $(\sigma'(\cdot), a'_{(\cdot)}, \eta'(\cdot))$ that satisfies the following conditions for every sender type $\theta \in \Theta$: (i) $\sigma'(\theta) \in \Pi^*$; (ii) $\hat{a}'_{\sigma(\theta)}$ maps each signal to itself; (iii) $(\sigma'(\theta), \hat{a}'_{\sigma'(\theta)})$ leads to the same joint distribution over the state ω and action a as $(\sigma(\theta), \hat{a}_{\sigma(\theta)})$; (iv) $p(\eta'(\sigma'(\theta))) = p(\eta(\sigma(\theta)))$.*

We begin by constructing $\sigma'(\cdot)$ as follows. For every $\theta \in \Theta$, we use $\pi'_\theta \in \Delta(\Omega \times A)$ to denote the joint distribution over the state ω and action a induced by the players' choices $\sigma(\theta)$ and $\hat{a}_{\sigma(\theta)}$ in the initial equilibrium. Set $\sigma'(\theta) = \pi'_\theta$ for all $\theta \in \Theta$. Next, we construct the decision rule $\hat{a}'_\pi$ following each choice of information structure π by the sender. When π is such that $\pi = \sigma'(\theta)$ for some $\theta \in \Theta$, $\hat{a}'_\pi$ is set to map every signal $s \in A$ to itself. For all other π, we let $\hat{a}'_\pi = \hat{a}_\pi$. Last, we construct the belief system $\eta'(\cdot)$. For all π such that $\pi = \sigma'(\theta)$ for some $\theta \in \Theta$, let $\eta'(\pi)$ be given by Bayes' rule. For all other π, we set $\eta'(\pi) = \eta(\pi)$.

By construction, the conjectured equilibrium $(\sigma'(\cdot), a'_{(\cdot)}, \eta'(\cdot))$ satisfies the conditions (ii) and (iii). It also satisfies condition (iv) by the following argument: Take any type $\theta \in \Theta$. If $\sigma'(\tilde{\theta}) = \sigma'(\theta)$ for some $\tilde{\theta} \neq \theta$, it must be that $p(\eta(\sigma(\tilde{\theta}))) = p(\eta(\sigma(\theta)))$ since otherwise either θ or $\tilde{\theta}$ would have incentives to deviate to the other's choice in the initial equilibrium. Thus, averaging over all such types $\tilde{\theta}$ that are pooled onto $\sigma'(\theta)$, we conclude that $p(\eta'(\sigma'(\theta))) = \mathbb{E}[\tilde{\theta} \,|\, \sigma'(\tilde{\theta}) = \sigma'(\theta)] = p(\eta(\sigma(\theta)))$.

Furthermore, the conjectured equilibrium actually constitutes a perfect Bayesian equilibrium: Consistency with Bayes' rule of $\eta'(\cdot)$ and sequential rationality of $\hat{a}'_\pi$ are inherited from their counterparts in the initial equilibrium. Sequential rationality of $\hat{a}'_\pi$ also implies that the obedience constraints are satisfied, thus the conjectured equilibrium fulfils condition (i). Additionally, for every sender type $\theta \in \Theta$, the material and image payoffs that she can secure from each choice of information structure remain the same as in the initial equilibrium. This equivalence in payoffs implies that she has no strict incentives to deviate from $\sigma(\theta)$ as otherwise $\sigma(\theta)$ would not have been optimal for her in the initial equilibrium.

The next two steps pertain the tie-breaking rule specified in the main text, i.e., the receiver always follows the sender's recommendation in case of indifference (even when it is off the equilibrium path). While it is possible to characterize all equilibria without specifying a tie-breaking rule on the receiver's side, doing so would require keeping track of the receiver's decision rule after every possible information structure by the sender off the equilibrium path. This would introduce a significant burden in terms of notation. In addition, with our chosen tie-breaking rule, the set of equilibira can be conveniently identified by solving the optimization problem (2).

However, one might question the extent to which our choice of tie-breaking rule is crucial for the analysis. To address this concern, we formally demonstrate that imposing our chosen tie-breaking rule does not result in any loss for the (on-path) equilibrium characterization, given a mild assumption on the players' payoffs.

To state the assumption formally, let $\mathcal{V}$ be the set of expected material payoffs of the sender arising from any information structure π of the sender and any sequentially rational decision a_π of the receiver given π, and by $\mathcal{W}$ the subset that arises from any information structure π for which the receiver has a uniquely optimal action after any signal realization and the unique sequentially rational decision rule of the receiver given π.

Assumption (G1). *The payoff environment of the sender-receiver game satisfies $\overline{\mathcal{V}} = \mathcal{W}$, where $\overline{\mathcal{V}}$ is the closure of $\mathcal{V}$.*

Intuitively, the assumption is useful for our purpose because it implies that information can be used to arbitrarily closely approximate the tie-breaking selection. Indeed, based on (G1), we can show that any equilibrium of the (more general) sender-receiver game considered in this appendix, which fulfills conditions (i) and (ii) outlined in Step 1, corresponds to an equilibrium of the game considered in the main text, with both equilibria coinciding on the equilibrium path, and vice versa.

Step 2. *Take any equilibrium $(\sigma(\cdot), a_{(\cdot)}, \eta(\cdot))$ of the (more general) sender-receiver game and suppose it satisfies the following conditions for every sender type $\theta \in \Theta$: (i) $\sigma(\theta) \in \Pi^*$ and (ii) $\hat{a}'_{\sigma(\theta)}$ maps each signal to itself. There is an equilibrium of the sender receiver-game considered in the main text that coincides with $(\sigma(\cdot), a_{(\cdot)}, \eta(\cdot))$ on the equilibrium path.*

To establish the claim, we first show that there is an equilibrium $(\sigma'(\cdot), a'_{(\cdot)}, \eta'(\cdot))$ of the general game for which (a) $\sigma' = \sigma$ and (b) for every $\pi \in \Pi^*$, the decision rule $\hat{a}'_\pi$ maps each signal to itself. Then, by disregarding the off-path decision rule a'_π and belief $\eta'(\pi)$ for any $\pi \notin \Pi^*$, the restriction of $(\sigma'(\cdot), a'_{(\cdot)}, \eta'(\cdot))$ yields an equilibrium of the game considered in the main text.

The new equilibrium has the same sender strategy as the initial one, $\sigma' = \sigma$. Also, for each $\pi \notin \Pi^*$, it has the same decision-rule, $a'_\pi = a_\pi$, and belief, $\eta'(\pi) = \eta(\pi)$. For any off-path information structure $\pi \in \Pi^*$, let a'_π be the decision rule that maps each signal to itself. To prove that $(\sigma'(\cdot), a'_{(\cdot)}, \eta'(\cdot))$ is an equilibrium, it suffices to show that no type prefers to choose any off-path information structure $\pi \in \Pi^*$ over $\pi(\theta)$ (since we did not modify anything else). To accomplish this, we calculate the expected material payoff of the sender generated by π and the decision rule that maps each signal to itself, denoted as V.

Condition (G1) ensures that there is a sequence of information structures $\pi_n \in \Pi^*$ such that, for any n, the receiver has a uniquely optimal action after any signal realization, and for which the expected material payoff of the sender generated by π_n and a_{π_n} converges to V. An application of the Bolzano-Weierstrass theorem shows that, without loss of generality, we can assume that the posterior means $p(\eta(\pi_n))$ converge. Choose $\eta(\pi)$ to be any belief distribution for which $p(\eta(\pi)) = \lim_{n\to\infty} p(\eta(\pi_n))$. If there were a sender type θ who strictly prefers to choose π over $\pi(\theta)$, then she would also strictly prefer to choose π_n over $\pi(\theta)$ for n large enough. However, this contradicts the assumption that $(\sigma(\cdot), a_{(\cdot)}, \eta(\cdot))$ is an equilibrium. Therefore, no sender type has a strict incentive to deviate to π.

Step 3. *Take any equilibrium $(\sigma(\cdot), a_{(\cdot)}, \eta(\cdot))$ of the sender receiver-game considered in the main text. There is an equilibrium in the (more general) sender-receiver game that coincides with $(\sigma(\cdot), a_{(\cdot)}, \eta(\cdot))$ on the equilibrium path.*

The new equilibrium maintains the same sender strategy as the initial one, with $\sigma' = \sigma$. Also, for each $\pi \notin \Pi^*$, it retains the same decision rule, $a'_\pi = a_\pi$, and belief, $\eta'(\pi) = \eta(\pi)$. For any off-path information structure $\pi \notin \Pi^*$, we specify a sequentially rational decision rule a'_π and a belief $\eta'(\pi)$ so that no type has strict incentives to deviate to such an information structure. Consider any sequentially rational decision rule a'_π of the receiver. Denote $\pi'' \in \Delta(\Omega \times A)$ the joint distribution over the state ω and action a induced by π and a'_π. Note that the decision rule $a'_{\pi''}$ that maps each signal to itself is sequentially rational given π'', thus it is adopted after π'' in the initial equilibrium given the tie-breaking assumption. Set $\eta'(\pi) = \eta(\pi'')$. Finally, since no type would want to deviate to π'' in the initial equilibrium, no type would want to deviate to π either given the choices a'_π and η'_π.

A.2 Formal Requirements of the D1 Criterion

Given a sender strategy $\sigma = \{\pi_\theta\}_{\theta\in\Theta}$ and the associated belief system $H = \{\eta(\pi)\}_{\pi\in\Pi}$ of the receiver, we define, for any $(\pi, \theta) \in \Pi^* \times \Theta$, two sets of beliefs as follows:

$$D^0(\pi,\theta) \equiv \{\tilde{\eta} \in \Delta(\Theta) : \mathbb{E}_\pi[v(s,\omega)] + \phi \cdot w(p(\tilde{\eta}),\theta) \geq \mathbb{E}_{\pi_\theta}[v(s,\omega)] + \phi \cdot w(p(\eta(\pi_\theta)),\theta)\}$$

and

$$D(\pi,\theta) \equiv \left\{\tilde{\eta} \in \Delta(\Theta) : \mathbb{E}_{\pi}[v(s,\omega)] + \phi \cdot w(p(\tilde{\eta}),\theta) > \mathbb{E}_{\pi_\theta}[v(s,\omega)] + \phi \cdot w(p(\eta(\pi_\theta)),\theta)\right\}.$$

A perfect Bayesian equilibrium with (σ, H) is selected by the D1 criterion if for any $\pi \in \Pi^*$ that is not used by any sender type under σ, and for all sender types θ and θ',

$$D^0(\pi,\theta) \subsetneq D(\pi,\theta') \implies \theta \notin supp(\eta(\pi)). \tag{14}$$

Note that incorporating the off-path choices $\pi \in \Pi \setminus \Pi^*$ into the above requirements will not alter the set of equilibria selected by the D1 criterion. This is because, given an equilibrium, for any $\pi \in \Pi \setminus \Pi^*$, there exists $\pi' \in \Pi^*$ that yields the same expected material payoff to the sender. Specifically, $\pi' \in \Delta(\Omega \times A)$ is given by the joint distribution of the state and the receiver action induced by π and decision rule of the receiver under π, as specified by the equilibrium. Hence, in the spirit of Banks and Sobel (1987) and Cho and Kreps (1987), the proposed equilibrium passes the test required by the D1 criterion at π if and only if it passes the test at π'.

A.3 The Single-Crossing Property

Lemma A1. *Take any two implementable material payoffs V, V' with $V > V'$ and any two receiver beliefs $\eta, \eta' \in \Delta(\Theta)$. If $\theta \in [0,1]$ is indifferent between (V,η) and (V',η'), then*

(a) all types $\theta' < \theta$ strictly prefer (V,η) over (V',η), and

(b) all types $\theta' > \theta$ strictly prefer (V',η') over (V,η).

Proof. Indifference of type θ means

$$V - V' = \phi \cdot [w(p(\eta'),\theta) - w(p(\eta),\theta)]. \tag{15}$$

Since $\partial w(p,\theta)/\partial p > 0$ and $V - V' > 0$, it is necessary that $p(\eta) < p(\eta')$. Then, given that $w(\cdot)$ has strictly increasing differences, the indifference condition (15) implies

$$V - V' > \phi \cdot [w(p(\eta'),\theta') - w(p(\eta),\theta')]$$

for all $\theta' < \theta$, and

$$V - V' < \phi \cdot [w(p(\eta'), \theta) - w(p(\eta), \theta)]$$

for all $\theta' > \theta$. □

A.4 Proof of Lemma 1

Let σ be an equilibrium strategy. Incentive compatibility implies that, for all sender types $\theta, \theta' \in \Theta$ with $\theta' < \theta$,

$$V(\theta; \sigma) + \phi \cdot w(p(\theta; \sigma), \theta) \geq V(\theta'; \sigma) + \phi \cdot w(p(\theta'; \sigma), \theta) \tag{16}$$

and

$$V(\theta'; \sigma) + \phi \cdot w(p(\theta'; \sigma), \theta') \geq V(\theta; \sigma) + \phi \cdot w(p(\theta; \sigma), \theta'). \tag{17}$$

Summing up (16) and (17), we obtain (with some rearrangement)

$$w(p(\theta; \sigma), \theta) - w(p(\theta'; \sigma), \theta) \geq w(p(\theta; \sigma), \theta') - w(p(\theta'; \sigma), \theta'). \tag{18}$$

Since $\theta > \theta'$ and $w(\cdot)$ has strictly increasing differences, (18) implies that $p(\theta; \sigma) \geq p(\theta'; \sigma)$. As the sender always prefers higher images, we also have $w(p(\theta; \sigma), \theta') \geq w(p(\theta'; \sigma), \theta')$. Hence, for (17) to hold it is necessary that $V(\theta; \sigma) \leq V(\theta'; \sigma)$. □

A.5 Proof of Lemma 2

Take an equilibrium with sender strategy σ and belief system H, and suppose that it satisfies D1. Suppose that there is some non-singleton $J \subseteq [0, 1]$ such that all types $\theta \in J$ choose the same $\pi \in \Pi^*$ with $\mathbb{E}_{\pi}[v(s, \omega)] = V > \underline{V}$. Take an information structure $\pi^{\varepsilon} \in \Pi^*$ that satisfies $\mathbb{E}_{\pi^{\varepsilon}}[v(s, \omega)] = V - \varepsilon$, which must exist for sufficiently small $\varepsilon > 0$ (see footnote 8).

Let $\mathbb{E}_{\eta(\pi^{\varepsilon})}[\tilde{\theta}]$ represent the receiver's posterior expectation about the sender's type upon observing the sender choosing the information structure π^{ε}, as induced by the belief system H. We argue that $\mathbb{E}_{\eta(\pi^{\varepsilon})}[\tilde{\theta}] \geq \sup J$ must hold. To prove this argument, we distinguish two cases. First, suppose that π^{ε} is a choice on the equilibrium path under the strategy σ, i.e., there exists $\theta \notin J$ such that $\sigma(\theta) = \pi^\varepsilon$. Then, by Lemma 1, we have $\theta \geq \sup J$. Since the choice of θ was arbitrary and the receiver's on-path beliefs must satisfy Bayes' rule, the claim $\mathbb{E}_{\eta(\pi^\varepsilon)}[\tilde{\theta}] = \pi^\varepsilon; \sigma] \geq \sup J$ immediately follows.

Second, suppose that no types will choose π^ε under the strategy σ. In this case, take any $\theta \in J$ with $\theta < \sup J$. Since the type distribution has full support, it holds $\sup J > \mathrm{E}[\tilde{\theta}|\tilde{\theta} \in J] > \inf J$. Therefore, there is $\epsilon > 0$ small enough so that, for the off-path belief $\mathbb{E}_{\eta(\pi^\varepsilon)}[\tilde{\theta}] = \sup J$, θ strictly prefers π^ϵ over π, and for $\mathbb{E}_{\eta(\pi^\varepsilon)}[\tilde{\theta}] = \inf J$, θ strictly prefers π over π^ϵ. An application of the intermediate value theorem shows that there must exist a posterior expectation $\hat{p} \in [0,1]$ such that if the receiver would hold a belief with this expectation and be obedient to the realization of the signal upon observing π^ε, then the type-θ sender would be indifferent between choosing π and π^ε. Moreover, given that $V - \varepsilon < V$, any sender with $\theta' < \theta$ would strictly prefer π to π^ε whenever type θ is being indifferent between these two pairs (Lemma A1). Hence, the D1 criterion requires that the receiver assigns zero weight to types $\theta' < \theta$ upon observing that π^ε was chosen by the sender. As a result, we have $\mathbb{E}_{\eta(\pi^\varepsilon)}[\tilde{\theta}] \geq \theta$. Since the choice of $\theta < \sup J$ was arbitrary, it again follows that the claim $\mathbb{E}_{\eta(\pi^\varepsilon)}[\tilde{\theta}] \geq \sup J$ must hold.

Next, we argue that for sufficiently small $\varepsilon > 0$, the expected utilities from π^ε will be strictly higher than that from π for all types $\theta \in J$. This is because, when $\varepsilon > 0$ is small enough, the following condition holds for all $\theta \in J$:

$$(V - \varepsilon) + \phi \cdot w\left(\mathbb{E}_{\eta(\pi^\varepsilon)}[\tilde{\theta}], \theta\right) \geq (V - \varepsilon) + \phi \cdot w\left(\sup J, \theta\right) > V + \phi \cdot w\left(\mathbb{E}[\tilde{\theta}|\tilde{\theta} \in J], \theta\right),$$

where the inequalities arise from the monotonicity property of $w(\cdot)$ and our previous conclusion that $\mathbb{E}_{\eta(\pi^\varepsilon)}[\tilde{\theta}] \geq \sup J > \mathbb{E}[\tilde{\theta}|\tilde{\theta} \in J]$. This contradicts the assumption that σ and H are the strategy and belief system of an equilibrium. □

A.6 Proof of Theorem 1: The If-Part

To establish the if-statement of Theorem 1, we verify that for any strategy $\sigma = \{\pi_\theta\}_{\theta \in \Theta}$ satisfying $\pi_\theta \in \Pi^*$ for all $\theta \in [0,1]$ and both conditions (i) and (ii), there exists a system of beliefs $H = \{\eta(\pi)\}_{\pi \in \Pi}$ of the receiver such that (σ, H) constitutes a D1 equilibrium. The construction of this belief system is as follows. For any $\pi \in \Pi^*$:

- If $\hat{\theta} = +\infty$ and $\mathbb{E}_\pi[v(s,\omega)] \geq \bar{V} - \phi \int_0^1 \frac{\partial w(x,x)}{\partial p} dx$, the receiver assigns probability one to the unique type $\theta \in [0,1]$ for which $\mathbb{E}_{\pi_\theta}[v(s,\omega)] = \mathbb{E}_\pi[v(s,\omega)]$.

- If $\hat{\theta} = +\infty$ and $\mathbb{E}_\pi[v(s,\omega)] < \bar{V} - \phi \int_0^1 \frac{\partial w(x,x)}{\partial p} dx$, the receiver assigns probability one to the type $\theta = 1$.
- If $\hat{\theta} < 1$ and $\mathbb{E}_\pi[v(s,\omega)] > \bar{V} - \phi \int_0^{\hat{\theta}} \frac{\partial w(x,x)}{\partial p} dx$, the receiver assigns probability one to the unique type $\theta \in [0,1]$ for which $\mathbb{E}_{\pi_\theta}[v(s,\omega)] = \mathbb{E}_\pi[v(s,\omega)]$.
- If $\hat{\theta} < 1$ and $\underline{V} < \mathbb{E}_\pi[v(s,\omega)] \leq \bar{V} - \phi \int_0^{\hat{\theta}} \frac{\partial w(x,x)}{\partial p} dx$, the receiver assigns probability one to the the type $\theta = \hat{\theta}$.
- If $\hat{\theta} < 1$ and $\mathbb{E}_\pi[v(s,\omega)] = \underline{V}$, the receiver updates his belief by restricting the type space to the subset $[\hat{\theta}, 1]$ and invoking Bayes' rule, which gives rise to the posterior expectation $\mathrm{E}_{\eta(\pi)}[\tilde{\theta}] = \mathrm{E}[\tilde{\theta} | \tilde{\theta} \in [\hat{\theta}, 1]]$.

Finally, the remaining out-of-equilibrium beliefs $\eta(\pi)$ for $\pi \in \Pi \setminus \Pi^*$ are determined by following the procedure: Take any sequentially rational decision rule of the receiver under π, i.e., any mapping $\hat{a}_\pi : \mathrm{supp}(\pi) \to \Delta(A)$ that maximizes the receiver's expected payoff for all signals realized from π. Let $\hat{\pi} \in \Delta(\Theta \times A)$ be the joint distribution of states and receiver actions as induced by π and $\hat{a}_\pi$. By construction, we have $\hat{\pi} \in \Pi^*$. Then, we complete the belief system by specifying $\eta(\pi) = \eta(\hat{\pi})$.

Sequential Rationality. Given the belief system H constructed earlier, we first establish that any information structure inducing a material payoff V with $\bar{V} - \phi \cdot \int_0^{\min\{\hat{\theta},1\}} \frac{\partial w(x,x)}{\partial p} dx > V > \underline{V}$ is strictly inferior for the sender. This is because she could increase the material payoff (by choosing an information structure that implements $V + \varepsilon$ for sufficiently small $\varepsilon > 0$) without altering the receiver's belief about her type.

Secondly, we argue that no type $\theta \in [0, \hat{\theta})$ can strictly benefit from choosing an information structure that induces a material payoff $V = \bar{V} - \phi \cdot \int_0^{\theta'} \frac{\partial w(x,x)}{\partial p} dx$ for some unique type $\theta' \in [0, \min\{\hat{\theta}, 1\}]$, which, given H, would lead to the receiver assigning probability one of her being type θ'. This is because, for all $\theta \in [0, \hat{\theta})$ and $\theta' \in [0, \min\{\hat{\theta}, 1\}]$, we have

$$\begin{aligned}
& V(\theta; \sigma) + \phi \cdot w(\theta, \theta) \\
= & V + \phi \cdot w(\theta', \theta) + [V(\theta; \sigma) - V] + \phi \cdot [w(\theta, \theta) - w(\theta', \theta)] \\
= & V + \phi \cdot w(\theta', \theta) + \phi \cdot \left[\int_0^{\theta'} \frac{\partial w(x, x)}{\partial p} dx - \int_0^{\theta} \frac{\partial w(x, x)}{\partial p} dx \right] - \phi \cdot \int_{\theta}^{\theta'} \frac{\partial w(x, \theta)}{\partial p} dx \\
= & V + \phi \cdot w(\theta', \theta) + \phi \cdot \int_{\theta}^{\theta'} \left[\frac{\partial w(x, x)}{\partial p} - \frac{\partial w(x, \theta)}{\partial p} \right] dx \\
\geq & V + \phi \cdot w(\theta', \theta),
\end{aligned} \tag{19}$$

where the second equality follows from condition (i) and the construction of V, and the inequality follows since $w(\cdot)$ has strictly increasing differences. Note that the inequality in (19) is strict whenever $\theta' \neq \theta$.

Thirdly, when $\hat{\theta} \in [0, 1)$, it is clear that no sender type in the pooling interval $[\hat{\theta}, 1]$ can alter her utility by choosing a different information structure that also leads to the minimal material payoff $\underline{V}$. Further, by construction, the cutoff type $\hat{\theta}$ is indifferent between pooling with higher types (by choosing some π that yields $\underline{V}$) and separating herself (by choosing some π that gives rise to $V = \bar{V} - \phi \int_0^{\hat{\theta}} \frac{\partial w(x,x)}{\partial p} dx$). Hence, Lemma A1 implies that the types in the separating interval $[0, \hat{\theta})$ cannot strictly benefit from deviating to an information structure that induces $\underline{V}$. Conversely, the types in the pooling interval $[\hat{\theta}, 1]$ would not have any strict incentive to deviate to an information structure that induces a material payoff $V = \bar{V} - \phi \cdot \int_0^{\theta'} \frac{\partial w(x,x)}{\partial p} dx$ for some unique type $\theta' \in [0, \min\{\hat{\theta}, 1\}]$.

D1 criterion. Take an off-path information structure $\pi' \in \Pi^*$. For any type θ, provided that $D^0(\pi', \theta)$ – the set of beliefs for which θ weakly prefers to deviate from her choice π_θ to π') is not empty – we define

$$\underline{p}(\pi', \theta) = \inf_{\eta \in D^0(\pi', \theta)} \mathbb{E}_\eta[\tilde{\theta}].$$

Note that since $\partial w(p, \theta)/\partial p > 0$, we have $\eta \in D^0(\pi', \theta) \iff \mathbb{E}_\eta[\tilde{\theta}] \geq \underline{p}(\pi', \theta)$ and $\eta \in D(\pi', \theta) \iff \mathbb{E}_\eta[\tilde{\theta}] > \underline{p}(\pi', \theta)$.

We distinguish two cases. First, suppose that there is $\theta \in [0, 1]$ such that $\mathbb{E}_{\pi'}[v(s, \omega)] = V(\theta; \sigma)$, which implies that $\underline{p}(\pi', \theta) = \mathbb{E}_{\eta(\pi_\theta)}[\tilde{\theta}]$. Consider any type θ' with $\pi_{\theta'} \neq \pi_\theta$. We have already shown that this type has *strict* incentives *not* to imitate type θ. This implies $\underline{p}(\pi',\theta') > \underline{p}(\pi',\theta)$, and therefore $D^0(\pi',\theta') \subsetneq D(\pi',\theta)$. Conversely, for any type θ'' with $\pi_{\theta''} = \pi_\theta$, clearly $\underline{p}(\pi',\theta'') = \underline{p}(\pi',\theta)$, and therefore $D^0(\pi',\theta'') = D^0(\pi',\theta) \supsetneq D(\pi',\theta)$. Thus, the D1 criterion requires that the receiver restricts his out-of-equilibrium belief to those types θ'' with $\pi_{\theta''} = \pi_\theta$. However, our belief system was just chosen this way.

Second, consider the case where no $\theta \in [0,1]$ exists such that $\mathbb{E}_{\pi'}[v(s,\omega)] = V(\theta;\sigma)$. If $\hat{\theta} = +\infty$, it is necessary that $\mathbb{E}_{\pi'}[v(s,\omega)] < V(1;\sigma)$. In this scenario, on-path incentive compatibility guarantees that $D^0(\pi',\theta) = \emptyset$ for all $\theta \in [0,1]$. As a result, the D1 criterion imposes no constraint on the out-of-equilibrium belief after observing π', so the belief system we constructed will clearly be consistent with the criterion.

Alternatively, if $\hat{\theta} \in [0,1)$, then it is necessary that

$$\underline{V} < \mathbb{E}_{\pi'}[v(s,\omega)] \leq \bar{V} - \phi \cdot \int_0^{\hat{\theta}} \frac{\partial w(x,x)}{\partial p} dx. \tag{20}$$

In this scenario, we claim that the following must hold for all $\theta < \hat{\theta}$:

$$\begin{aligned} V(\theta;\sigma) + \phi \cdot w(\theta,\theta) > & \bar{V} - \phi \cdot \int_0^{\hat{\theta}} \frac{\partial w(x,x)}{\partial p} dx + \phi \cdot w(\hat{\theta},\theta) \\ > & \mathbb{E}_{\pi'}[v(s,\omega)] + \phi \cdot w\left(\underline{p}(\pi',\hat{\theta}),\theta\right). \end{aligned} \tag{21}$$

The first inequality follows from (19). As for the second inequality, it holds because the following reason: it holds because of the following reason: By construction, the cut-off type $\hat{\theta}$ is indifferent between joining the pool by inducing the minimum material payoff $\underline{V}$ and separating herself by inducing $V = \bar{V} - \phi \int_0^{\hat{\theta}} \frac{\partial w(x,x)}{\partial p} dx$. But then, type $\hat{\theta}$ should also be indifferent to choosing π' if it leads the receiver to hold the posterior expectation $\underline{p}(\pi',\hat{\theta})$. Thus, (20) and the single-crossing property established in Lemma A1 jointly imply that, between the two pairs of material payoffs and images, $(\bar{V} - \phi \int_0^{\hat{\theta}} \frac{\partial w(x,x)}{\partial p} dx, \hat{\theta})$ and $(\mathbb{E}_{\pi'}[v(s,\omega)], \underline{p}(\pi',\hat{\theta}))$, type θ must strictly prefer the former to the latter.

Given (21), we can assert that $\underline{p}(\pi',\theta) > \underline{p}(\pi',\hat{\theta})$ for all $\theta < \hat{\theta}$. Further, since

$$\underline{V} + \phi \cdot w\left(\mathbb{E}[\tilde{\theta}|\tilde{\theta} \geq \hat{\theta}], \hat{\theta}\right) = \mathbb{E}_{\pi'}[v(s,\omega)] + \phi \cdot w\left(\underline{p}(\pi',\hat{\theta}),\hat{\theta}\right),$$

Lemma A1 and (20) jointly imply that

$$\underline{V} + \phi \cdot w\left(\mathbb{E}[\tilde{\theta}|\tilde{\theta} \geq \hat{\theta}], \theta\right) > \mathbb{E}_{\pi'}[v(s,\omega)] + \phi \cdot w\left(\underline{p}(\pi', \hat{\theta}), \theta\right)$$

for all $\theta > \hat{\theta}$. As a result, we also have $\underline{p}(\pi', \theta) > \underline{p}(\pi', \hat{\theta})$ for all $\theta > \hat{\theta}$. In sum, we can conclude that $D^0(\pi', \theta) \subsetneq D(\pi', \hat{\theta})$ for all $\theta \neq \hat{\theta}$, so the D1 criterion requires that the receiver assigns probability one to type $\hat{\theta}$ when he observes π'. However, our belief system was just chosen this way. □

A.7 Proof of Theorem 2

Part (i): Consider any action a_0 that is optimal for the receiver under no information. Let π^N be the information structure that consistently sends the signal $s = a_0$ to the receiver (which effectively provides no information about the state to the receiver). Evidently, $\pi^N \in \Pi^*$, so the material payoffs that it yields to the sender and the receiver are given by $V^N \equiv \mathbb{E}_{\pi^N}[v(s,\omega)] = \mathbb{E}_{\mu_0}[v(a_0,\omega)]$ and $\underline{U}$, respectively. By assumption, U^* is unique and satisfies $U^* = \bar{U} > \underline{U}$. Consequently, we must infer that $\bar{V} > V^N$, as otherwise, π^N would be optimal for the sender in the pure persuasion benchmark, leading to the contradiction that $\underline{U} = \bar{U}$. Take an arbitrary D1 equilibrium strategy $\sigma = \{\pi_\theta\}_{\theta \in [0,1]}$. For every type $\theta \in (0, \hat{\theta})$, recall that her expected payoff $V(\theta; \sigma)$ will be uniquely pinned down by the envelope formula (5). Therefore, there must exist a non-empty interval $(0, \check{\theta}) \subseteq (0, \hat{\theta})$ such that $V(\theta;\sigma) \geq V^N$ holds for all $\theta \in (0, \check{\theta})$.

Now, let $\bar{\pi}$ be an information structure that yields the expected payoff $\bar{V}$ to the sender. For each $\theta \in (0, \check{\theta})$, consider the following information structure $\check{\pi}_\theta$: conditional on each state, with probability $\lambda(\theta) = (V(\theta;\sigma) - V^N)/(\bar{V} - V^N) < 1$, the receiver observes a signal s drawn according to $\bar{\pi}$; with the remaining probability $1 - \lambda(\theta)$, the signal is generated according to π^N. It is straightforward to verify that $\check{\pi}_\theta \in \Pi^*$, $\mathbb{E}_{\check{\pi}_\theta}[v(s,\omega)] = V(\theta;\sigma)$, and

$$\mathbb{E}_{\check{\pi}_\theta}[u(s,\omega)] = \lambda(\theta) \cdot \bar{U} + (1 - \lambda(\theta)) \cdot \underline{U} < \bar{U}. \tag{22}$$

Next, define a strategy $\check{\sigma}$ for the sender as follows: for all $\theta \in (0, \hat{\theta})$, let $\check{\sigma}(\theta) = \check{\pi}_\theta$; for all other θ, let $\check{\sigma}(\theta) = \sigma(\theta)$. By Theorem 1, $\check{\sigma}$ is part of a D1 equilibrium. Moreover, since the type distribution is continuous and has full support, (22) implies that the ex-ante expected payoff of the receiver must be strictly lower than U^*, meaning that he is harmed by the presence of the sender's image concerns.

Part (ii): Let $\sigma = \{\pi_\theta\}_{\theta\in[0,1]}$ be the sender's strategy in a Pareto-optimal D1 equilibrium. Assume, for the sake of contradiction, that the receiver's expected payoff is not decreasing everywhere. Then, there must exist $\theta, \theta' \in [0, 1]$ such that $\theta < \theta'$ and

$$\mathbb{E}_{\pi_\theta}[u(s, \omega)] < \mathbb{E}_{\pi_{\theta'}}[u(s, \omega)] \leq \bar{U}. \tag{23}$$

If $V(\theta; \sigma) = V(\theta'; \sigma)$, we could simply ask type θ to adopt the same information structure as type θ', thereby increasing the welfare of the receiver without affecting the sender's. Hence, without loss of generality, we may focus on the scenario $\bar{V} \geq V(\theta; \sigma) > V(\theta'; \sigma)$. Now consider the following information structure $\breve{\pi}_\theta$: conditional on each state, with probability $\lambda = (V(\theta; \sigma) - V(\theta'; \sigma))/(\bar{V} - V(\theta'; \sigma)) \in [0, 1]$, the information structure generates a signal s according to $\bar{\pi}$; with the remaining probability $1 - \lambda$, the signal is generated according to $\pi_{\theta'}$. It is straightforward to check that $\breve{\pi}_\theta \in \Pi^*$, $\mathbb{E}_{\breve{\pi}_\theta}[v(s, \omega)] = V(\theta; \sigma)$, and

$$\mathbb{E}_{\breve{\pi}_\theta}[u(s, \omega)] = \lambda \cdot \bar{U} + (1 - \lambda) \cdot \mathbb{E}_{\pi_{\theta'}}[u(s, \omega)] > \mathbb{E}_{\pi_\theta}[u(s, \omega)]. \tag{24}$$

Therefore, it is possible to construct a D1 equilibrium strategy $\breve{\sigma}$ that always yields a weakly higher payoff to the receiver than σ, and this payoff difference will even be strict when the sender's type is θ. Hence, the strategy σ cannot be Pareto-optimal if the associated payoff for the receiver is not decreasing everywhere within the interval $[0, 1]$.

Part (iii): To prove quasi-convexity, it suffices to demonstrate that the receiver's payoff is either monotonically decreasing or U-shaped with respect to the sender's type. Let $\sigma = \{\pi_\theta\}_{\theta\in[0,1]}$ be the sender's strategy in a Pareto-worst D1 equilibrium. Define V^N_{min} and V^N_{max} as the minimum and maximum material payoffs that the sender may obtain when the receiver acts under no information, respectively. Note that these two values may differ because if there are several prior-optimal actions for the receiver, there is some flexibility for the information structure to determine which of these actions will be taken by the receiver. Let π^N_{min} and π^N_{max} be the information structures that lead to these two material payoffs for the sender, respectively. Also, let $\theta^N_{min} = \sup\{\theta \in [0, 1] : V(\theta; \sigma) \geq V^N_{max}\}$ and $\theta^N_{max} = \inf\{\theta \in [0, 1] : V(\theta; \sigma) \leq V^N_{min}\}$.

First, we argue that the receiver's expected payoff must be decreasing everywhere on $[0, \theta^N_{min}]$. To prove this, suppose by contradiction that there exist $\theta, \theta' \in [0, \theta^N_{min}]$ such that

$\theta < \theta'$ and

$$\underline{U} \le \mathbb{E}_{\pi_\theta}[u(s,\omega)] < \mathbb{E}_{\pi_{\theta'}}[u(s,\omega)]. \tag{25}$$

If $V(\theta;\sigma) = V(\theta';\sigma)$, we could simply ask type θ' to adopt the same information structure as type θ, which clearly decreases the welfare of the receiver without affecting the sender's. Hence, without loss of generality, we may focus on the scenario $V(\theta;\sigma) > V(\theta';\sigma) \ge V^N_{max}$. Next, consider the following information structure $\breve{\pi}_{\theta'}$: conditional on each state, with probability $\lambda' = (V(\theta';\sigma) - V^N_{max})/(V(\theta;\sigma) - V^N_{max}) \in [0,1]$, the information structure generates a signal s according to π_θ; with the remaining probability $1-\lambda'$, the signal is generated according to π^N_{max}. It is straightforward to check that $\breve{\pi}_{\theta'} \in \Pi^*$, $\mathbb{E}_{\breve{\pi}_{\theta'}}[v(s,\omega)] = V(\theta';\sigma)$, and

$$\mathbb{E}_{\breve{\pi}_{\theta'}}[u(s,\omega)] = \lambda' \cdot \mathbb{E}_{\pi_\theta}[u(s,\omega)] + (1-\lambda') \cdot \underline{U} < \mathbb{E}_{\pi_{\theta'}}[u(s,\omega)]. \tag{26}$$

Therefore, it is possible to construct a D1 equilibrium strategy $\breve{\sigma}$ that always gives the receiver a weakly lower payoff to the receiver than σ, and this payoff difference will even be strict when the sender's type is θ'. Hence, the receiver's expected payoff $\mathbb{E}_{\breve{\pi}_\theta}[u(s,\omega)]$ must be monotonically decreasing in θ within the interval $[0, \theta^N_{min}]$.

Second, for types $\theta \in [\theta^N_{min}, \theta^N_{max}]$, we claim that the receiver's payoff is constant in θ in the Pareto-worst equilibrium. To see this, note that we can replace the information structure of each sender type $\theta \in [\theta^N_{min}, \theta^N_{max}]$ with one that mixes over π^N_{min} and π^N_{max}, ensuring that the sender's material payoff remains $V(\theta;\sigma)$. At the same time, the receiver's payoff under such information structures is $\underline{U}$, since he will merely be mixing over his prior-optimal actions. Therefore, in the Pareto-worst equilibrium, the receiver's payoff will stay constant at $\underline{U}$ within the interval $[\theta^N_{min}, \theta^N_{max}]$.

Lastly, we argue that the receiver's expected payoff must be increasing everywhere on $[\theta^N_{max}, 1]$. Suppose not. Then, there must exist $\theta, \theta' \in [\theta^N_{max}, 1]$ such that $\theta < \theta'$ and

$$\underline{U} \le \mathbb{E}_{\pi_{\theta'}}[u(s,\omega)] < \mathbb{E}_{\pi_\theta}[u(s,\omega)]. \tag{27}$$

If $V(\theta;\sigma) = V(\theta';\sigma)$, it would be feasible to have type θ adopt the same information structure as type θ', which clearly decreases the welfare of the receiver without altering the sender's. Hence, without loss, we may focus on the scenario $V(\theta';\sigma) < V(\theta;\sigma) \le V^N_{min}$. Using a similar construction of "grand" information structures involving π^N_{min} as before, it can be shown that there exists a D1 equilibrium strategy that always gives a weakly lower payoff to the receiver than σ, and this payoff difference will even be strict when the sender's type is θ. Hence, the receiver's expected payoff must be increasing on $[\theta^N_{max}, 1]$. This concludes our proof of the receiver's expected payoff being quasi-convex in the whole interval $[0, 1]$. □

A.8 Results and Proofs Related to the Examples

A.8.1 The Utility-Frontier with Almost-Perfectly-Aligned Preferences

In the following, we provide formal details regarding the second instance of congruent preferences (Example 1) that we discussed in the main text. Formally, we specify $A = \Omega = \{-1, 0, 1\}$ and define the material payoff functions for the players as follows:

$$u(a,\omega) = \begin{cases} 1 & \text{if } a = \omega, \\ 0 & \text{otherwise,} \end{cases} \quad \text{and} \quad v(a,\omega) = \begin{cases} 1 & \text{if } a = \omega, \\ 0 & \text{if } a \neq \omega \text{ and } a \neq -1, \\ -1 & \text{if } a \neq \omega \text{ and } a = -1. \end{cases} \tag{28}$$

The interpretation of the above payoff specification is that the material interests of the players are *almost* perfectly aligned. Both players would like to match the action with the true state. However, the action $a = -1$ is somewhat riskier than others for the sender, because she will be additionally punished when the receiver selects it by mistake. By contrast, the receiver is indifferent between different types of errors. Despite the different payoff functions, the congruency condition (11) is met since the players agree on the first-best action in each state.

Let the prior distribution μ_0 be such that $\Pr(\omega = 1) = \Pr(\omega = 0) = 0.4$ and $\Pr(\omega = -1) = 0.2$. It is clear that $\bar{V} = \bar{U} = 1$ and $\underline{U} = 0.4$. To solve $\underline{V}$, first note that the sender's expected material payoff depends mainly on two things: (i) the total probability that the receiver will take the right action, denoted as $\Pr(a = \omega)$; (ii) the total probability that the receiver will wrongly take the action $a = -1$, denoted as $\Pr(a = -1|\omega \neq -1)$. Regardless of which information structure $\pi \in \Pi^*$ is used by the sender, it is necessary that $\Pr(a = \omega) \geq 0.4$, because the receiver cannot do strictly worse than sticking to his prior-optimal action. At the same time, an upper bound for $\Pr(a = -1|\omega \neq -1)$ is 0.5: If $\Pr(a = -1|\omega \neq -1) > 0.5$, the receiver would necessarily hold a posterior with $\Pr(\omega = -1|s = -1) < 1/3$, which means that it cannot be rational for him to take the recommended action -1.

Now consider an information structure $\underline{\pi}$ which recommends the action $a = -1$ with probability one in state $\omega = -1$, and it recommends $a = 1$ or $a = -1$ with equal probabilities in the other two states. It can be checked that $\underline{\pi} \in \Pi^*$ and that $\underline{\pi}$ achieves the two aforementioned bounds on the receiver's decision-making probabilities simultaneously. Since the sender is worse off when the receiver less often takes the right action and more often chooses the action $a = -1$ in the wrong states, $\underline{\pi}$ must give the lowest possible payoff to the sender among all information structures, thus $\underline{V} = \mathbb{E}_{\underline{\pi}}[v(s, \omega)] = 0$.

Given the analysis above, we deduce that the set of implementable payoff profiles, formally defined as $\mathcal{W} = \{(V, U) : \exists\, \pi \in \Pi^* \text{ such that } V = \mathbb{E}_\pi[v(s, w)] \text{ and } U = \mathbb{E}_\pi[u(s, w)]\}$, must fall within the rectangle $[\underline{V}, \bar{V}] \times [\underline{U}, \bar{U}] = [0, 1] \times [0.4, 1]$. In addition, $\mathcal{W}$ is closed and convex (Zhong, 2018). Hence, to characterize $\mathcal{W}$, it suffices to answer the following question: For a given level of the receiver's payoff $U \in [\underline{U}, \bar{U}]$, what are the maximum and minimum material payoffs that the sender can achieve by using some information structure $\pi \in \Pi^*$, respectively? Note that the receiver's expected payoff equals exactly the ex-ante probability that he takes the right action. Hence, the question boils down to identifying the set of $\Pr(a = -1|\omega \neq -1)$ values that the sender can induce without violating the requirement $\Pr(a = \omega) = U$.

For $U \in [0.4, 0.6]$, the previous upper bound on $\Pr(a = -1|\omega \neq -1)$ can still be achieved. This is made possible by the information structure $\underline{\pi}^U \in \Pi^*$ characterized by the following conditional probabilities (of recommending different actions in different states): $\underline{\pi}^U(-1|-1) = 1$, $\underline{\pi}^U(1|1) = \underline{\pi}^U(-1|1) = \underline{\pi}^U(-1|0) = 0.5$, $\underline{\pi}^U(0|0) = (U - 0.4)/0.4$, and $\underline{\pi}^U(1|0) = (0.8 - U)/0.4$. The resulting payoff to the sender, $U - 0.4$, is the minimal one across all information structures that induce the receiver to choose $a = \omega$ with probability U. Consequently, for all $U \in [0.4, 0.6]$, $(U - 0.4, U)$ is on the boundary of $\mathcal{W}$, which corresponds to a point on the red curve (below the kink) in Panel (b) of Figure 2. As for $U \in (0.6, 1]$, the highest probability that the receiver will wrongly choose $a = -1$ becomes $1 - U$. This (revised) upper bound can be achieved by an information structure $\underline{\pi}^U \in \Pi^*$ with $\underline{\pi}^U(-1| - 1) = 1$, $\underline{\pi}^U(1|1) = \underline{\pi}^U(0|0) = (U - 0.2)/0.8$, and $\underline{\pi}^U(-1|1) = \underline{\pi}^U(-1|0) = (1 - U)/0.8$. Thus, for each $U \in (0.6, 1]$, $(2U - 1, U)$ is on the boundary of $\mathcal{W}$, and it corresponds to a point on the red curve (this time above the kink) in the figure.

Finally, for all $U \in [0.4, 1]$, the maximum material payoff of the sender is achieved when the receiver *never* chooses $a = -1$ in states $\omega \in \{0, 1\}$. Hence, every payoff profile (U, U) with $U \in [0.4, 1]$ is in the boundary of $\mathcal{W}$. In addition, since both $(0, 0.4)$ and $(0.4, 0.4)$ are implementable and $\underline{U} = 0.4$, any payoff profile $(V, 0.4)$ with $V \in (0, 0.4)$ is also a boundary point of $\mathcal{W}$ given the convexity of $\mathcal{W}$. Taken together, we obtain the blue curve depicted in the figure.

A.8.2 Transforming the Quadratic-Loss Games

Consider the quadratic-loss game that we discussed in Examples 2 and 4. For any $\pi \in \Pi^*$, the obedience constraints of the receiver imply $s = \mathbb{E}_\pi[\omega'|s]$. As a result, the expected material loss of a type-θ sender is

$$\begin{aligned}
&\mathbb{E}_\pi\left[(s - a^*(\omega,\theta))^2\right] \\
=&\mathbb{E}_\pi\left[(\mathbb{E}_\pi[\omega'|s])^2\right] + \mathbb{E}_\pi\left[(a^*(\omega,\theta))^2\right] - 2\mathbb{E}_\pi\left[\mathbb{E}_\pi[\omega'|s]\cdot(f(\theta)\cdot\omega + g(\theta))\right] \\
=&\mathbb{E}_\pi\left[(\mathbb{E}_\pi[\omega'|s])^2\right] + \mathbb{E}_{\mu_0}\left[(a^*(\omega,\theta))^2\right] - 2f(\theta)\cdot\mathbb{E}_\pi\left[(\mathbb{E}_\pi[\omega'|s])^2\right] - 2g(\theta)\cdot\mathbb{E}_{\mu_0}[\omega] \\
=&(1-2f(\theta))\cdot\mathbb{E}_\pi[\mathbb{E}_\pi[\omega'|s]^2] + K(\theta),
\end{aligned}$$

where the outer expectation is taken with respect to ω and s (but not ω'). The second equality follows from several arguments. First, the law of iterated expectation yields $\mathbb{E}_\pi\left[\mathbb{E}_\pi[\omega'|s]\right] = \mathbb{E}_{\mu_0}[\omega]$, where we denote by μ_0 the prior belief about the state. Second, we have

$$\mathbb{E}_\pi\left[\mathbb{E}_\pi[\omega'|s]\cdot\omega\right] = \mathbb{E}_\pi[\mathbb{E}_\pi[\mathbb{E}_\pi[\omega'|s]\cdot\omega|s]] = \mathbb{E}_\pi[\mathbb{E}_\pi[\omega'|s]^2] = \mathbb{E}_\pi[\mathbb{E}_\pi(\omega'|s)^2],$$

where the first step is due to Fubini's theorem, which says that we can integrate sequentially, first over ω and then over s. The second step follows from linearity of the expected value, and the third step is valid since the function to be integrated does not depend on ω. For the third equality above, we use $K(\theta) \equiv \mathbb{E}_{\mu_0}\left[(a^*(\omega,\theta))^2\right] - 2g(\theta)\cdot\mathbb{E}_{\mu_0}[\omega]$ to collect all the (θ-specific) constant terms. A similar calculation shows that $\mathbb{E}_\pi\left[(s-\omega)^2\right] = -\mathbb{E}_\pi[\mathbb{E}_\pi[\omega'|s]^2] + \mathbb{E}_{\mu_0}[\omega^2]$.

Now, suppose that $f(\theta) > 0.5$ for all $\theta \in [0,1]$ and compare the following two utility functions of the sender, given by a material and an image payoff function each: $-(a - a^*(\omega,\theta))^2$ and $w(p(\eta),\theta)$, versus $\hat{v}(a,\omega) = -(a-\omega)^2$ and $\hat{w}(p(\eta),\theta) = w(p(\eta),\theta)/(2f(\theta)-1)$. We claim that these two specifications lead to the same preference over the pairs $(\pi,\eta) \in \Pi^* \times \Delta(\Theta)$, for each type $\theta \in [0,1]$. This is because, for all θ and all such (π, η) and (π', η'), we have

$$\begin{aligned}
&\mathbb{E}_{\pi}\left[-(s-a^*(\omega,\theta))^2\right] + \phi \cdot w(p(\eta),\theta) \geq \mathbb{E}_{\pi'}\left[-(s-a^*(\omega,\theta))^2\right] + \phi \cdot w(p(\eta'),\theta) \\
\iff &(2f(\theta)-1) \cdot \left[\mathbb{E}_{\pi}[\mathbb{E}_{\pi}[\omega'|s]^2] - \mathbb{E}_{\pi'}[\mathbb{E}_{\pi'}[\omega'|s]^2]\right] + \phi \cdot [w(p(\eta),\theta) - w(p(\eta'),\theta)] \geq 0 \\
\iff &\mathbb{E}_{\pi}[\mathbb{E}_{\pi}[\omega'|s]^2] - \mathbb{E}_{\pi'}[\mathbb{E}_{\pi'}[\omega'|s]^2] + \phi \cdot [\hat{w}(p(\eta),\theta) - \hat{w}(p(\eta'),\theta)] \geq 0 \\
\iff &\mathbb{E}_{\pi}[\hat{v}(s,\omega)] + \phi \cdot \hat{w}(p(\eta),\theta) \geq \mathbb{E}_{\pi'}[\hat{v}(s,\omega)] + \phi \cdot \hat{w}(p(\eta'),\theta).
\end{aligned}$$

Hence, under the current parametric assumption, the quadratic-loss game in Example 2 has the same equilibrium set as a game where the receiver's utility function remains unchanged, but the sender's utility function is instead given by $\hat{v}(\cdot)$ and $\hat{w}(\cdot)$.

Similarly, if $f(\theta) < 0.5$ for all $\theta \in [0,1]$, then, as described in Example 4, we effectively have a quadratic loss game where the sender's material payoff function is given by $\hat{v}(a,\omega) = (a-\omega)^2$, while her image payoff function is given by $\hat{w}(p(\eta),\theta) = w(p(\eta),\theta)/(1-2f(\theta))$.

A.8.3 Receiver-Optimality in Example 5

Consider our first example of state-independent sender preferences (Example 5), where both the state and the action spaces are binary. Recall the information structure $\bar{\pi}^q$, which is defined according to (12) for each $q \in [0, 2\mu_0]$. We argue that, among all information structures that induce the receiver to choose the high action with probability q, $\bar{\pi}^q$ is the one that gives the highest payoff to the receiver.

Take any information structure $\pi \in \Pi^*$ that recommends the high action with the unconditional probability q, and let $\pi(a|\omega)$ be the conditional probability that it recommends action a when the state is ω. Then, it is necessary that

$$\mu_0 \cdot \pi(1|1) + (1-\mu_0) \cdot \pi(1|0) = q. \tag{29}$$

Therefore, the receiver's expected utility under π is given by

$$\begin{aligned}
\mathbb{E}_{\pi}[u(s,\omega)] &= \mu_0 \cdot \pi(1|1) + (1-\mu_0) \cdot \pi(0|0) \\
&= q - (1-\mu_0) \cdot \pi(1|0) + (1-\mu_0) \cdot (1-\pi(1|0)) \\
&= 1 - \mu_0 + q - 2(1-\mu_0) \cdot \pi(1|0).
\end{aligned}$$

Recall that the information structure $\bar{\pi}^q$ satisfies $\bar{\pi}^q(1|0) = 0$ when $q \in [0, \mu_0]$, so we see that it is receiver-optimal in this case. At the same time, note that, using (29), the receiver's expected utility can also be written as

$$\mathbb{E}_\pi[u(s, \omega)] = 1 + (2\pi(1|1) - 1) \cdot \mu_0 - q.$$

Recall that the information structure $\bar{\pi}^q$ satisfies $\bar{\pi}^q(1|1) = 1$ when $q \in (\mu_0, 2\mu_0]$. Thus, $\bar{\pi}^q$ is also receiver-optimal in this case.

A.8.4 Pareto-Extremal Equilibria in Example 6

Consider the setting specified in Example 6, and suppose the state ω is uniformly distributed on $[0, 1]$. Since $v(a, \omega) = a$, the sender's expected material payoff equals the probability of the receiver taking action $a = 1$. Clearly, the minimum of this probability is 0, which the sender can achieve by providing no information. Below we prove that the maximum of this probability is $2 - 2\underline{u}$.

Take any information structure $\pi \in \Pi^*$, and let $\pi(1|\omega)$ be the probability that it generates the signal $s = 1$. We argue that, for the purpose of maximizing the sender's material payoff, it is without loss of generality to focus on π with increasing $\pi(1|\cdot)$. To see this, consider any $\pi \in \Pi^*$ and suppose $\pi(1|\omega) > \pi(1|\omega')$ for some ω, ω' with $\omega < \omega'$. Then, one can construct a new information structure $\hat{\pi}$ by swapping the distributions of the recommended action for ω and ω'. Given that $\omega < \omega'$, this relaxes the obedience constraints (which require $\mathbb{E}[\omega|s = 1] \geq \underline{u}$ and $\mathbb{E}[\omega|s = 0] \leq \underline{u}$) and preserves the overall probability of $a = 1$ since we are only changing the information structure on a measure zero of states; so it gives the same sender payoff.

Next, take any $\pi \in \Pi^*$ such that $\pi(1|\cdot)$ is increasing. Let $\hat{\omega} = \inf\{\omega : \pi(1|\omega) = 1\}$. We argue that if $\hat{\omega} > 2\underline{u} - 1$, then π cannot be maximizing the sender's material payoff. Without loss of generality, suppose that $\mathbb{E}[\omega|s = 1] = \underline{u}$ under π. Together with $\hat{\omega} > 2\underline{u} - 1$, this implies that the probability of getting the signal $s = 1$ is strictly positive when the state lies in $[0, 2\underline{u} - 1)$. But then, one can reassign this probability mass to the interval $[2\underline{u} - 1, \hat{\omega})$ without violating the incentive compatibility constraints. In fact, doing so will relax the previously binding constraint $\mathbb{E}[\omega|s = 1] = \underline{u}$, meaning that it would become feasible for the sender to induce the receiver to take action $a = 1$ with even higher probability.

In sum, the above arguments demonstrate that the preceding arguments show that the sender can attain her maximum material payoff using the following information structure, which generates a deterministic signal s contingent on the true state: $s = 1$ if $\omega \geq 2\underline{u} - 1$, and $s = 0$ otherwise. Under this information structure, the total probability of the receiver choosing action $a = 1$ is $2 - 2\underline{u}$.

Lastly, we show that how one can use two simple classes of "interval disclosure" information structures to describe the entire Pareto-frontier of the equilibrium set. For this purpose, we define two different information structures for every $q \in [0, 2 - 2\underline{u}]$. The first, denoted as $\bar{\pi}^q$, generates a deterministic signal s contingent on the true state: $s = 1$ if $\omega \geq 1 - q$, and $s = 0$ otherwise. The second information structure, denoted as $\underline{\pi}^q$, also follows a deterministic signal-generating rule: $s = 1$ if $\omega \in [\underline{u} - q/2, \underline{u} + q/2]$, and $s = 0$ otherwise. It can be verified that both $\bar{\pi}^q$ and $\underline{\pi}^q$ can induce the receiver to choose the non-default action $a = 1$ with a probability of q. Furthermore, $\bar{\pi}^q$ ($\underline{\pi}^q$) yields the highest (lowest) expected payoff to the receiver among all information structures that implement the same marginal distribution of actions: For instance, if an information structure $\pi \in \Pi^*$, which induces $\Pr(a = 1) = q$ like $\bar{\pi}^q$, advises the receiver to choose $a = 0$ with some positive probability when $\omega \geq 1 - q$, it must also suggest $a = 1$ in certain situations when $\omega < 1 - q$. By exchanging these two recommendations, we can create an information structure that increases the receiver's payoff while keeping his total probability of choosing $a = 1$ unchanged. Hence, similar to Example 5, there exists a Pareto-optimal (Pareto-worst) D1 equilibrium in which each type θ chooses the information structure $\bar{\pi}^{q(\theta)}$ ($\underline{\pi}^{q(\theta)}$), where $q(\theta)$ represents the probability that type θ would induce the receiver to take action $a = 1$.

A.9 Proof of Theorem 4

Take the information structure $\pi^N \in \Pi^*$ that convey no information about the state to the receiver (e.g., one that consistently generates a signal $s = a_0 \in A$ regardless of the state, where a_0 is any action that maximizes the receiver's expected payoff under his prior μ_0). Take any information structure $\pi^F \in \Pi^*$ that allows the receiver to achieve his full-information payoff (e.g., one that generates a signal $s = a^*(\omega) \in A$ conditional on each state $\omega \in \Omega$, where $a^*(\omega)$ is any action that maximizes the receiver's payoff when the true state is ω). Let V^N and V^F be the expected material payoffs under π^N and π^F, respectively. Since the receiver's payoff U^* in the pure persuasion benchmark satisfies $U^* \in (\underline{U}, \bar{U})$, it is necessary that $\bar{V} > \max\{V^N, V^F\}$.

Otherwise, providing full or no information would have been optimal in the pure-persuasion benchmark, leading to the contradiction $U^* \in \{\bar{U}, \underline{U}\}$.

If ϕ is sufficiently small, all D1 equilibria will be fully separating and the material payoff $V(1;\sigma)$ that the highest type would choose to implement will be sufficiently close to $\bar{V}$. As a result, $V(1;\sigma) > \max\{V^N, V^F\}$ irrespective of the which D1 equilibrium strategy σ is selected. In particular, there exists a D1 equilibrium in which each type θ uses a "grand" information structure that mixes appropriately between $\bar{\pi}$, the sender-optimal information structure $\bar{\pi}$ absent image concerns, and π^N. Clearly, the receiver is strictly worse off in this equilibrium relative to the equilibrium in the pure persuasion setting (i.e., when $\phi = 0$). Similarly, there also exists a D1 equilibrium in which the sender's strategy is always a combination of $\bar{\pi}$ and π^F. Clearly, the receiver must be strictly better off in this equilibrium relative to the equilibrium in the pure persuasion setting. □

A.10 Additional Details of the Application in Section 4.3

A.10.1 Micro-Founding the Monotonicity Assumption in Section 4.3

In this subsection, we provide a setting of electoral competition which endogenizes the key assumption in Section 4.3, namely, that the ratio $(w_2(\cdot) - w_3(\cdot))/w_1(\cdot)$ is strictly increasing. We suppose that, in the third stage, the incumbent politician described in Section 4.3 – who we now call candidate A – competes with another candidate B (challenger) for an election. Each candidate $j = A, B$ has a private type $\theta_j \in [0, 1]$, which is independently distributed with a mean denoted by $\bar{\theta}_j$.

The candidate who wins the election will get to choose a policy $y \in \mathbb{R}$. The voters have a common preference over policies, represented by $-|y - y^*|$. For each candidate $j = A, B$, with probability θ_j, she will have the same policy preference as the voters. With the remaining probability $1 - \theta_j$, the candidate's preference will be $-|y - (y^* + 1)|$. Thus, the higher θ_j (i.e., the less corrupt the candidate), the more likely that the candidate will act perfectly according to the voters' interest once elected.

Recall that the voters may update their belief about candidate A's type upon observing the latter's choices in previous stages (whereas the belief about candidate B is given by the prior). Let ϵ be the common preference shock that directly adds to each voter's utility whenever A is elected, and p is the public posterior about A's type. It is straightforward to show that voters would support candidate A if and only if $\epsilon \geq \bar{\theta}_B - p$. The winning probability of candidate A is then given by $1 - G\left(\bar{\theta}_B - p\right)$.

Overall, for candidate A, her expected payoff from the electoral competition is

$$G\left(\bar{\theta}_B - p\right) \cdot [-\theta_A(1-\bar{\theta}_B) - (1-\theta_A)\bar{\theta}_B] = -G\left(\bar{\theta}_B - p\right) \cdot (w_2(\theta_A) - w_3(\theta_A)),$$

where $w_2(\theta_A) = 0$ and $w_3(\theta_A) = -\theta_A - \bar{\theta}_B + 2\bar{\theta}_B\theta_A$ are the expected payoffs upon winning and losing the election, respectively. Given that $w_1(\cdot)$ is strictly decreasing, it can be checked that $(w_2(\cdot) - w_3(\cdot))/w_1(\cdot)$ is strictly increasing in θ_A whenever $\bar{\theta}_B < 0.5$ is additionally satisfied.

A.10.2 Reduced-Form Description of the Equilibrium of the Dynamic Game

We explain that the dynamic game as in 4.3 has a reduced-form description in terms of the model in Section 2. We begin by arguing that in any equilibrium, the politician will always disclose the result of the study, regardless of whether it is positive about the reform or not.

To see this, let Θ_π be the set of types that choose the same study π in an equilibrium. Note that disclosing a result s with $\pi(s|1)/\pi(s|0) \geq \ell_0 \equiv (1-\mu_0)/\mu_0$ will for sure lead to the adoption of the reform. This implies that, for a type $\theta \in \Theta_\pi$ to prefer keeping this result private, non-disclosure must lead to a higher image payoff than disclosing s. In this game, the analogue of the single-crossing property holds for bundles of expected material payoffs attached to strategies of the sender (which are given by the choice of study and subsequent disclosure choices) and the receiver's beliefs about the sender type. This way, the set Θ_π partitions into two (potentially empty) and disjoint sets Θ_π^- and Θ_π^+ so that all types in Θ_π^- are strictly smaller than all types in Θ_π^+ and do not disclose s, while all types in Θ_π^+ disclose s. If Θ_π^- is non-empty, then the ordering of the sets Θ_π^- and Θ_π^+ implies that non-disclosure leads to a strictly lower image payoff as well as a weakly lower material payoff than disclosure. This cannot be in equilibrium, so that Θ_π^- must be empty. Therefore, following the classic "unraveling" argument (Grossman, 1981; Milgrom, 1981), in any equilibrium, all results s with $\pi(s|1)/\pi(s|0) \geq \ell_0$ will necessarily be disclosed. An analogous argument establishes that any result s with $\pi(s|1)/\pi(s|0) < \ell_0$ will also be disclosed.

Given that the politician would always disclose what she learns from the study, (on the equilibrium path) the voters' posterior belief about the politician's type would only depend on the chosen study. Consequently, a type-θ politician obtains the following payoff from choosing a study π:

$$\Pr(\pi(s|1)/\pi(s|0) \geq \ell_0) \cdot w_1(\theta) + (1 - G(\underline{u} - \alpha p)) \cdot w_2(\theta) + G(\underline{u} - \alpha p) \cdot w_3(\theta), \tag{30}$$

where $p = \mathbb{E}_{\eta(\pi)}[\theta]$. Without loss of generality, suppose that each politician type chooses a study that only gives rise to a binary result – either positive ($s = 1$) or negative ($s = 0$) about the reform. Naturally, disclosing the positive result is equivalent to making a recommendation to pass the reform, while disclosing the negative result is the same as recommending to maintain the status quo. Thus, for each type of politician, maximizing (30) is equivalent to

$$\max_{\pi} \Pr(s = 1 \,|\, \pi) + \frac{w_2(\theta)}{w_1(\theta)} - G(\underline{u} - \alpha p) \cdot \frac{w_2(\theta) - w_3(\theta)}{w_1(\theta)},$$

subject to the constraints that $\pi(1|\omega), \pi(0|\omega) \in [0, 1]$ and $\pi(0|\omega) + \pi(1|\omega) = 1 \; \forall \omega \in \{0, 1\}$, and $\pi(1|1)/\pi(1/0) \geq \ell_0$. Thus, the equilibrium problem of the current application maps into our setting by specializing Example 5 of Section 3.2 with

$$u(a, \omega) = \begin{cases} 0 & \text{if } a = 0 \\ 1 & \text{if } a = 1 \text{ and } \omega = 1 \\ -1 & \text{if } a = 1 \text{ and } \omega = 0 \end{cases} \tag{31}$$

and

$$v(a, \omega) = \mathbb{1}_{a=1}, \; \phi = 1, \; w(p, \theta) = \frac{w_2(\theta)}{w_1(\theta)} - G(\underline{u} - \alpha p) \cdot \frac{w_2(\theta) - w_3(\theta)}{w_1(\theta)}.$$

(Note here that in Example 5 we described the receiver's utilities as $u(a, \omega) = \mathbb{1}_{a=\omega}$. However, both this utility specification and (31) lead to the same best response of the receiver as a function of the belief over the state ω.)

A.10.3 Non-Monotone Welfare Effects of Populism

We argue that a change in α can have a non-monotone effect on the welfare of the public. For this purpose, we parameterize the reduced-form version of the dynamic game with $w_2(\theta) = 0$, $w_3(\theta)/w_1(\theta) = -\theta - 1$, and ϵ is uniformly distributed on $[0, 1]$. Assume also that $1 > \underline{u} > \alpha$

always holds. Taken together, we have

$$w(p,\theta) = -(\underline{u} - \alpha p) \cdot (\theta + 1) \quad \text{and} \quad \frac{\partial w(p,\theta)}{\partial p} = \alpha(\theta + 1).$$

It then follows from Theorem 1 and our analysis of Example 5 that in any D1 equilibrium, all types θ below a unique cutoff $\hat{\theta}$ are separating, and each $\theta < \hat{\theta}$ will commission a study that implements the reform with the following probability:

$$q(\theta) = 2\mu_0 - \int_0^\theta \alpha(x+1)dx = 2\mu_0 - \alpha\left(\frac{\theta^2}{2} + \theta\right).$$

In contrast, all types $\theta \geq \hat{\theta}$ will opt for a completely uninformative study, in which case the reform will for sure be rejected by the public. Additionally, it can be verified that

- $\hat{\theta} = +\infty$ when $\alpha < 4\mu_0/3$;
- $\hat{\theta} \in (0,1)$ and is strictly decreasing in α when $4\mu_0/3 \leq \alpha < 2\mu_0/\mathbb{E}_\Gamma[\tilde{\theta}]$;
- $\hat{\theta} = 0$ when $\alpha \geq 2\mu_0/\mathbb{E}_\Gamma[\tilde{\theta}]$.

Now consider the Pareto-optimal equilibrium which, as shown in Example 5, can be sustained by using the family of information structures (12). When α is sufficiently small, the equilibrium is fully separating, and higher types provide more information about the reform to the public. In this case, a local increase in α will benefit the public by incentivizing the politician to be even more honest. However, a large α could hurt the public by incentivizing the politician to provide less information (when some types are already recommending against reforms that are beneficial) and/or hindering the learning of the politician's true type (when pooling occurs in equilibrium).

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