On Political Information and Culture *

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We revisit a standard model of costly information acquisition and aggregation in large majority elections: An electorate of $2n+1$ voters have common preferences but are uncertain about which of two policies is better and may acquire costly private information before voting (Martinelli, 2006). Two novel insights emerge: First, political information (or informedness) can exhibit complements: if others acquire more information, this may increase a voter's own incentive to become informed. We characterize when, in our setting, political information exhibits substitutes and when it exhibits complements. Second, the complementarities have efficiency consequences: Voters may coordinate on a high-information equilibrium that is efficient (as $n \to \infty$) but may also coordinate on an inefficient low-information equilibrium. We discuss implications for voter-information initiatives, and the importance of a culture of political informedness.

The Condorcet jury theorem and its modern versions (e.g. Austen-Smith and Banks, 1996; Feddersen and Pesendorfer, 1997) represent a central efficiency benchmark for democratic institutions: Even when relevant political information is dispersed among a large number of voters, majority elections lead to efficient outcomes under broad conditions. Piketty (1999) interprets this result as a political analog of the First Welfare Theorem for markets.

A challenge to the efficiency of political institutions is that political information is typically costly: In a large election, an individual voter has a negligible probability of affecting the outcome, so that she may acquire too little information when information is costly, or even no information at all, precluding efficiency. This is the rational-ignorance problem emphasized by Downs (1957).

Martinelli (2006) made a fundamental observation about this problem, clarifying the conditions under which a version of the Condorcet jury theorem persists and "rational ignorance" does not preclude efficiency. Although each voter's private

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incentive to acquire information becomes small in a large electorate, the election may still aggregate many small pieces of information. His analysis formalizes the horse race between the growing number of voters and the shrinking amount of information acquired by each individual voter. If the marginal cost of arbitrarily imprecise information becomes arbitrarily close to zero, there exists an equilibrium sequence in which the election outcome is asymptotically efficient. Subsequent work extends and refines this positive message in related environments (see, e.g., Martinelli, 2007; Triossi, 2013; Oliveros, 2013).

This paper adds to the discussion by revisiting the setting of Martinelli (2006). The rational-ignorance discussion focuses on the public-good , i.e. the strategic substitute nature of political information, stressing that voters underinvest into political information because they can free-ride on the investments of others.

Our first result qualifies this view and highlights that political information (or informedness) can also exhibit strategic complementarities, in the sense that local changes in how much information other voters acquire may increase a given voter's incentives to acquire information herself. We characterize when political information exhibits substitutes and when it exhibits complements (Proposition 1).

The efficiency implication of this observation is substantial. Our second result shows that efficient equilibrium sequences almost always coexist with non-trivial inefficient ones in which information aggregation is only partial, even when the marginal cost of imprecise information is arbitrarily low (e.g. costs that vanish exponentially as the precision x goes to zero, such as $C(x) = \mathrm{e}^{-\frac{1}{x}}$). Voters may thus miscoordinate on an inefficient equilibrium. This means that the literature's costly-information version of the Condorcet jury theorem should be read as a possibility result, where efficiency relies on equilibrium coordination. The condition for the inefficient equilibrium is generic: Voters must not be exactly indifferent between the two election alternatives under the prior (Theorem 1).

The coexistence of a low-information and a high-information equilibrium is driven by the strategic complementarities of political information. In contrast, for the (non-generic) parameters of the model where political information is purely a substitute (as in the main specification in Martinelli (2006)), equilibria with information acquisition are unique, and coordination is not an issue.

Following Schelling (1980) and Myerson (2009), we interpret equilibrium coordination as a reduced-form representation of political culture. Societies may differ in the conventions, expectations, and focal points that make one equilibrium rather than another salient. In this sense, the low- and high-information equilibria correspond to different cultures of political informedness.

Empirical work documents corresponding differences in "cultures of news consumption." News consumption varies not only with individual preferences and information costs, but also with contextual norms (Toff and Kalogeropoulos, 2020). For example, some citizens follow political news because they regard keeping informed as part of civic duty (McCombs and Poindexter, 1983; Palmer and Toff, 2020). By contrast, other citizens follow a "news-finds-me" culture: they expect important political information to reach them through social media, peers, or everyday exposure without active search (Gil de Zúñiga *et al.*, 2017, 2020). These findings suggest that political informedness is shaped partly by shared norms about whether citizens should actively follow politics. Moreover, the findings suggest that social media may be associated with more passive information norms.

This perspective also offers a possible interpretation of the limited effects of standard voter-information campaigns, as documented in the meta-analysis of (Dunning *et al.*, 2019). Our model delivers an explanation: if information provision affects individual information costs but leaves citizens' expectations about others' information acquisition and surrounding norms largely unchanged, equilibrium still predicts low information acquisition. Voter-information initiatives may therefore need to do more than provide facts or lower individual information costs: they may also need to foster a culture of political informedness, in which active engagement with political information is expected.

Related Literature. The paper contributes to a larger literature on voting with costly information acquisition. The main contribution is to highlight that political information is not a pure substitute, in a strategic setting. Instead, political information can exhibit strategic-complement features, and we show that this has sharp efficiency implications in terms of a coordination problem. The previous literature has focused solely on the substitutes aspect of political information, driven by the free-riding incentives that arise when information costs are borne privately by individual voters.

In terms of the model, the paper is closest to the literature surrounding the Condorcet jury theorem, in particular, the literature mentioned above following Martinelli (2006)'s model. Mukhopadhaya (2003); Koriyama and Szentes (2009) also explore the Condorcet jury model and show that larger than optimal committee sizes do lead to social welfare losses relative to smaller committee sizes due to the free-riding motive, but that these losses need not be large.

Other related strands of the literature have explored other aspects. For example, Persico (2004); Gershkov and Szentes (2009); Gerardi and Yariv (2008) explore questions of the design of optimal voting mechanisms and committees. Since this literature typically focuses on the most efficient equilibrium, our results motivate the question under which conditions other non-trivial equilibria exist in these settings, possibly due to complementarities of political information. In such instances, questions of robust mechanism design naturally arise: For example, which voting mechanisms are worst-case optimal across all (non-trivial) equilibria?

There is also an experimental literature that has evaluated testable implications of the theories, see, e.g. Grosser and Seebauer (2016); Elbittar *et al.* (2020); Mechtenberg and Tyran (2019); Bhattacharya *et al.* (2017). Our analysis invites follow-up work that tests the extent to which coordination problems matter empirically when voter information is endogenous.

The coexistence of a low-information and a high-information equilibrium is reminiscent of the low- and high-turnout equilibria in costly-voting models such as Palfrey and Rosenthal (1983). That literature shows that the high-turnout equilibria can be fragile to strategic uncertainty about preferences, costs, or the number of voters (Palfrey and Rosenthal, 1985; Myerson, 1998). A companion paper studies related information-acquisition games with strategic uncertainty about preferences, information costs, and prior beliefs, and shows that the same coordination problem persists (Heese, 2022). Thus, the multiplicities that arise under costly information and costly participation are substantially different.

1 Model

We restate the model of Martinelli (2006) in its original notation. There are $2n+1 \geq 3$ voters, two policies A and B, and a binary state $z \in \{z_A, z_B\}$. The voters hold a common prior. The prior probability of state z_A is $q_A \in (0,1)$, and the prior probability of state z_B is $q_B = 1 - q_A$. Voters receive utility $U(d,z)$ if the outcome is d and the state is z. We denote $U(A,z_A) - U(B,z_A) = r_A$ and $U(B,z_B) - U(A,z_B) = r_B$ and assume that $r_A, r_B > 0$. Thus, all voters prefer A in state z_A and B in z_B. We assume $q_A r_A > q_B r_B$, so that policy A maximizes expected utility under the prior.[1]

The timing is as follows. Each voter chooses the precision $x \in [0, 1/2]$ of a binary private signal $s \in \{s_A, s_B\}$, where

$$\frac{1}{2} + x = \Pr(s_A \mid z_A) = \Pr(s_B \mid z_B).$$

When choosing precision x, the voter bears cost $C(x)$. Then, the state and private signals realize. After observing their private signals, all voters vote simultaneously, either for A or for B. Finally, the outcome is decided by simple majority rule.

We assume that C is strictly increasing, strictly convex, twice continuously differentiable on $(0, \frac{1}{2})$ and that $C(0) = 0$; in other words, acquiring no information is costless.

A pure strategy is a triple (x, v_A, v_B), where $x \in [0, 1/2]$ specifies the chosen precision, v_A specifies which policy the voter votes for after observing signal s_A, and v_B specifies which policy the voter votes for after observing signal s_B. A mixed strategy α is a probability distribution over pure strategies. We consider Bayes–Nash equilibria in mixed and symmetric strategies across the voters. An equilibrium in which some strategy (x, v_A, v_B) with $x > 0$ is in the support is called an *equilibrium with information acquisition.* Oftentimes, we abbreviate and simply write *equilibrium.*

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[1] The case $q_A r_A < q_B r_B$ yields qualitatively the same results with the roles of A and B reversed. In the knife-edge case $q_A r_A = q_B r_B$, the low-information equilibrium sequence identified in the main result (Theorem 1) does not exist.

2 Equilibrium Representation

We provide a compact characterization of equilibria with information acquisition, for large electorates. To state the result, we first explain the "pivotal calculus" of strategic voters.

Fix a voter and the strategy α of all other voters. Whenever the votes of the other voters do not split into n votes for A and n votes for B, the fixed voter's choice of her strategy does not affect the outcome. Thus, when comparing expected utilities from two strategies, it suffices to consider the pivotal event in which the votes of the other voters split into n votes for A and n votes for B. Denote by $\pi(z;\alpha)$ the probability that a voter votes for A in state $z \in \{z_A, z_B\}$ under α. The likelihood of the pivotal event is

$$\Pr(\text{piv} \mid z; \alpha) = \binom{2n}{n} \Big(\pi(z;\alpha)(1-\pi(z;\alpha))\Big)^n. \tag{1}$$

Our result characterizes equilibria as strategies $\alpha(x,\delta)$ that mix between (x, A, B) with probability $1-\delta$ and $(0, A, A)$ with probability $\delta > 0$ and satisfy two relevant equilibrium conditions.

Lemma 1 *Assume $q_A r_A > q_B r_B$ and $C'(0) = 0$. For all sufficiently large n, a strategy α is an equilibrium with information acquisition if and only if there is a pair $(x,\delta) \in (0, \frac{1}{2}) \times (0,1)$ so that $\alpha = \alpha(x,\delta)$ and α satisfies*

$$\Big(\Pr(\text{piv} \mid z_A;\alpha) q_A r_A + \Pr(\text{piv} \mid z_B;\alpha) q_B r_B\Big) \left(\frac{1}{2} + x\right) - C(x) = \Pr(\text{piv} \mid z_A;\alpha) q_A r_A, \tag{2}$$

$$\Pr(\text{piv} \mid z_A; \alpha(x,\delta)) q_A r_A + \Pr(\text{piv} \mid z_B; \alpha(x,\delta)) q_B r_B = C'(x). \tag{3}$$

The formal proof relies heavily on arguments already present in Martinelli (2006) and is relegated to the appendix. The first equilibrium condition states that voters are indifferent between (x, A, B) and $(0, A, A)$. Formally, compare the expected utilities from both strategies. Up to the constant utility from non-pivotal events, the expected utility from choosing (x, A, B) is

$$\begin{aligned}&\Pr(\text{piv} \mid z_A;\alpha) q_A \Big(U(A, z_A)\left(\frac{1}{2}+x\right) + U(B, z_A)\left(\frac{1}{2}-x\right)\Big)\\ &\quad + \Pr(\text{piv} \mid z_B;\alpha) q_B \Big(U(B, z_B)\left(\frac{1}{2}+x\right) + U(A, z_B)\left(\frac{1}{2}-x\right)\Big) - C(x).\end{aligned}$$

The expected utility from $(0, A, A)$ is, up to the same constant,

$$\Pr(\text{piv} \mid z_A; \alpha) q_A U(A, z_A) + \Pr(\text{piv} \mid z_B; \alpha) q_B U(A, z_B).$$

Subtracting $\Pr(\text{piv} \mid z_A; \alpha) q_A U(B, z_A) + \Pr(\text{piv} \mid z_B; \alpha) q_B U(A, z_B)$ from both expressions yields the stated indifference condition.

The second equilibrium condition states that the marginal cost of acquiring precision $x > 0$ equals the marginal benefit. Equivalently, it is the first-order condition for the optimal precision.

Under the mixed strategy $\alpha(x, \delta)$, a voter chooses (x, A, B) with probability $1-\delta$ and $(0, A, A)$ with probability δ. The induced voting probabilities are

$$\begin{aligned} \pi(z_A; \alpha(x, \delta)) &= (1-\delta)\left(\frac{1}{2} + x\right) + \delta, \\ \pi(z_B; \alpha(x, \delta)) &= (1-\delta)\left(\frac{1}{2} - x\right) + \delta. \end{aligned} \tag{4}$$

Note that the prior bias toward A given by $q_A r_A > q_B r_B$ matters for the qualitative nature of equilibria. It implies that all equilibria are shifted toward voting for A in the sense that all uninformed voters choose A. A prior bias toward B, i.e. $q_A r_A < q_B r_B$, would imply that, in all equilibria with information acquisition, the uninformed voters choose B. Thus, these equilibria involve mixing between (x, A, B) and $(0, B, B)$ instead of mixing between (x, A, B) and $(0, A, A)$.

In the following, we identify equilibria α with information acquisition with their representing pairs (x, δ).

3 Complements and Substitutes

The usual rational-ignorance discussion of the literature (e.g., in Downs (1957)) focuses on the public-good, i.e., strategic-substitute, nature of political information, stressing that voters underinvest in political information because they can free-ride on the investments of others.

We qualify this discussion by showing that political information exhibits strategic substitute effects but also strategic complementarity effects. Formally, we say that a strategy $\alpha(x, \delta)$ exhibits *strategic complements (substitutes)* in information acquisition if the *marginal benefit of information* is locally increasing in x, i.e. if

$$\frac{\partial \mathrm{MB}_n(x,\delta)}{\partial x} > (<)\ 0$$

for $\mathrm{MB}_n(x,\delta) = \Pr(\text{piv} \mid z_A; \alpha(x,\delta)) q_A r_A + \Pr(\text{piv} \mid z_B; \alpha(x,\delta)) q_B r_B$.

The next result characterizes which strategies exhibit complements and which substitutes.

Proposition 1 *Suppose $C'(0) = 0$ and $q_A r_A > q_B r_B$, and consider the voters' strategy $\alpha(x,\delta)$. For every sufficiently large n, there exists a threshold $\delta_{1,n} \in (0, 1/2)$ such that*

1. *for $0 \leq \delta < \delta_{1,n}$, the voter's strategy exhibits substitutes for all $x \in (0, \frac{1}{2})$;*
2. *for $\delta_{1,n} < \delta < 1/2$, the voter's strategy exhibits substitutes, then complements, and then again substitutes, when varying x from 0 to $\frac{1}{2}$;*
3. *for $1/2 < \delta < 1$, the voter's strategy exhibits substitutes and then complements, when varying x from 0 to $\frac{1}{2}$.*

The proof is in the appendix. The marginal benefit of information is best understood through its connection to the *margin of victory* in the two states,

$$\begin{aligned} MV(z; x, \delta) &= \left| \pi(z; \alpha(x,\delta)) - \frac{1}{2} \right| \\ &= \begin{cases} \delta + (1-\delta)\left(\frac{1}{2} + x\right) - \frac{1}{2} & \text{for } z = z_A, \\ \left| \delta + (1-\delta)\left(\frac{1}{2} - x\right) - \frac{1}{2} \right| & \text{for } z = z_B. \end{cases} \end{aligned}$$

The marginal benefit increases in the pivotal likelihoods $\Pr(\text{piv}|z; \alpha(x,\delta))$, which are themselves monotone decreasing in the margin of victory, as

$$\Pr(\text{piv} \mid z; \alpha(x,\delta)) = \binom{2n}{n} \left(\frac{1}{4} - MV(z; x, \delta)^2 \right)^n .$$

Overall, a larger margin of victory implies a smaller marginal benefit of information.

We explain the three regimes now via this connection to the margins of victory.

When $\delta = 0$, the margin of victory increases in x in both states. Thus, the value of information is monotonically decreasing, corresponding to substitutes. This monotonicity extends to values of δ in a whole interval $[0, \delta_{1,n})$, by continuity.

When $\delta_{1,n} < \delta < 1$, the vote share in z_A is larger than $\frac{1}{2}$ for all x but the vote share in z_B is larger than $\frac{1}{2}$ only if

$$x < x'' \text{ for } x'' = \frac{\delta}{2(1-\delta)}. \tag{5}$$

For $x < x''$, an increase in x has two countervailing effects: it increases the margin of victory in z_A, but it decreases it in z_B. The proof clarifies that the first effect dominates on an interval $[0, x']$, where $0 < x' < \min\{\frac{1}{2}, x''\}$, and the second effect dominates on $[x', \min\{\frac{1}{2}, x''\}]$. If $\delta > \frac{1}{2}$, then $x'' > \frac{1}{2}$ and these two intervals imply the overall substitutes-complements pattern. If $\delta < \frac{1}{2}$, there is another interval $[x'', \frac{1}{2}]$ on which the margin of victory increases with x in both states, implying an overall substitutes-complements-substitutes pattern.

Figure 1 below illustrates the three regimes in an example.

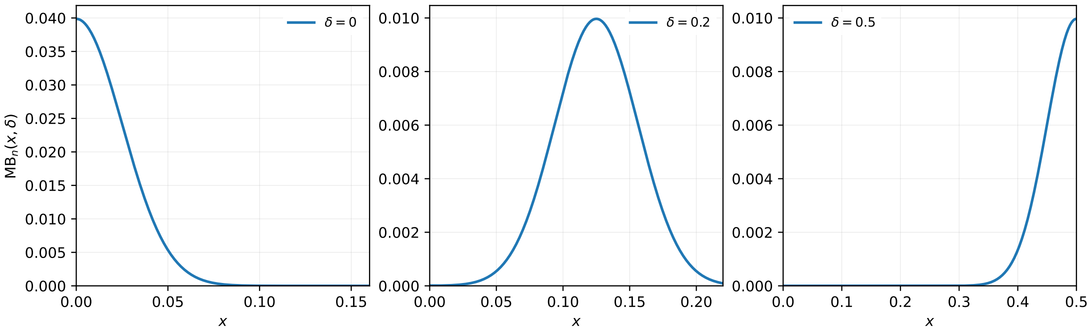

Figure 1: The marginal benefit $\text{MB}_n(x, \delta)$ of information as a function of $x \in (0, \frac{1}{2})$ for fixed $\delta = 0$ (left), $\delta = 0.2$ (middle), and $\delta = 0.5$ (right), in an example with $n = 200$, $C(x) = x^3$, $r_A = r_B = 1$, $q_A = 3/4$, and $q_B = 1/4$.

4 The Voters' Coordination Problem

We characterize the outcomes of large elections ($n \to \infty$). An equilibrium sequence $(\alpha_n)_{n\in\mathbb{N}}$ has *asymptotically efficient outcomes* if the probability that A is elected in state z_A and the probability that B is elected in z_B both converge to one as $n \to \infty$. It has *asymptotically inefficient outcomes* otherwise.

We focus on situations in which efficiency is possible. As shown by Martinelli (2006), a sequence of equilibria with asymptotically efficient outcomes exists if and only if $C'(0) = C''(0) = 0$. This condition captures that arbitrarily imprecise information has arbitrarily low marginal cost. In the efficient sequence, individual information acquisition becomes arbitrarily imprecise, but slowly enough for the aggregate information—acquired by an increasing number of voters—to enable asymptotically efficient outcomes. Under the same condition, aggregate information costs converge to zero. So, the equilibrium sequence maximizes utilitarian welfare of the agents, even if costs are taken into account (cf. Theorem 3 and Theorem 4 in Martinelli (2006)).

We show that, whenever efficiency is possible, the voters face a coordination problem. They may miscoordinate on an equilibrium sequence with asymptotically inefficient outcomes. In the inefficient equilibria, individual information acquisition is so low that information aggregation fails; that is, an outside observer cannot learn the state from observing the realized vote share, as $n \to \infty$.[2]

The next result provides a complete characterization of all equilibrium sequences.

Theorem 1 *Assume* $C'(0) = C''(0) = 0$.

1. *There exists a sequence of equilibria with asymptotically efficient outcomes.*
2. *There exists a sequence of equilibria with information acquisition that has asymptotically inefficient outcomes. Along this sequence, the prior-favored policy A is elected with probability converging to one in both states.*
3. *Any equilibrium sequence with information acquisition and convergent outcome probabilities has one of two limit outcomes: either it is asymptotically*

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[2] A small ambiguity in the statement of Theorem 4 (i) in Martinelli (2006) is relevant for the interpretation of our result. Theorem 4 (i) can be read either as a claim about the asymptotic behavior of all equilibrium sequences with information acquisition, or more narrowly as a claim about the particular equilibrium sequence constructed in the proof. Our result, therefore, clarifies that Theorem 4 (i) needs to be interpreted as an existence result, not as a classification of all equilibria with information acquisition. The stronger reading of Theorem 4 (i) fails because inefficient equilibrium sequences also exist.

efficient, or the prior-favored policy A is elected with probability converging to one in both states.

The formal proof of Theorem 1 is in the appendix. It constructs the equilibrium sequences using a novel fixed-point theorem, a generalization of the Poincaré–Miranda theorem, which is established in a companion paper, Ekmekci *et al.* (2025).[3] Economically, what drives the coexistence of the low- and high-information equilibria are the complementarities of political information described in Proposition 1.

To get some intuition, it is instructive to compare the "asymmetric" setting considered here, in which A is the prior-favored policy, with the "symmetric" main setting in Martinelli (2006) in which the voters are exactly indifferent given the prior. In the symmetric setting, equilibrium incentives are governed by a pure substitutes logic and no coordination problem arises, unlike in the asymmetric setting.

To explain this, we describe equilibria in both settings via a simple one-dimensional fixed-point equation and then highlight the role of the complementarities with an example illustration. In the asymmetric setting, for each $x \in (0, \frac{1}{2})$, denote by $\delta(x)$ the minimal δ solving the indifference condition (2) and by $\hat{x}(x, \delta(x))$ the precision solving the first-order condition (3) given $\alpha(x, \delta(x))$. Any fixed point

$$x = \hat{x}(x, \delta(x)) \tag{6}$$

of the correspondence $x \mapsto \hat{x}(x, \delta(x))$ induces a strategy $\alpha(x, \delta(x))$ satisfying both equilibrium conditions (2) and (3), hence an equilibrium. In the symmetric setting, the symmetry implies that all equilibria satisfy $\delta = 0$ and are characterized by the first-order condition (3), which we rewrite as

$$x = \hat{x}(x, 0); \tag{7}$$

cf. Theorem 1 in Martinelli (2006).

Figure 2 illustrates the two fixed-point equations (6) and (7) now in an example. The right panel shows the mapping $x \to \hat{x}(x, 0)$ in the symmetric setting and the left panel shows the mapping $x \to \hat{x}(x, \delta(x))$ in the asymmetric setting.

In the symmetric setting, $\delta = 0$ implies that political information is a substitute (cf. Proposition 1). As a consequence, the precision $\hat{x}(x, 0)$ of the best response

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[3] The result in Ekmekci *et al.* (2025) generalizes the original Poincaré–Miranda theorem in (Miranda, 1940).

to $\alpha(x,0)$ is strictly decreasing in x: for a given voter, the more information the other voters acquire, the less incentive the voter has to acquire information herself. The monotonicity implies a unique crossing point with the identity map and thus a unique equilibrium. Absent complementarities, equilibrium is unique.

In the asymmetric setting, the complementarities imply a non-monotone shape of the fixed-point mapping $x \to \hat{x}(x, \delta(x))$. This enables two crossing points with the identity mapping, and thus two equilibria. At the high-information equilibrium, voters acquire more information but also have stronger incentives to acquire information under the best response, compared to the low-information equilibrium.

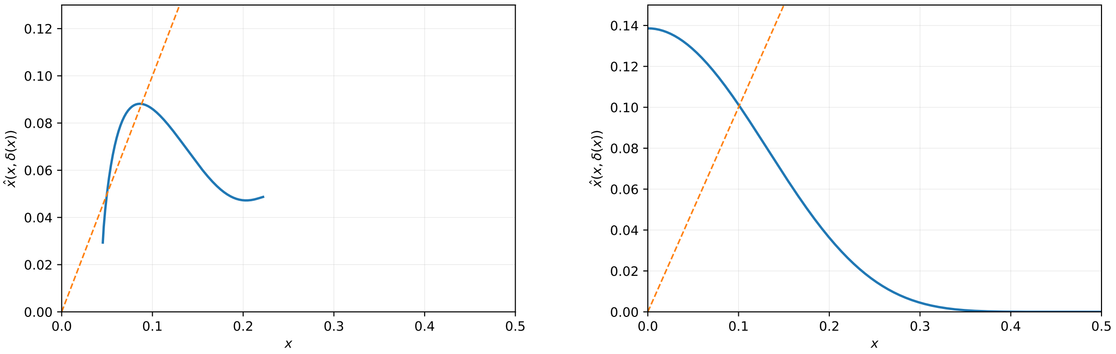

Figure 2: Example with $n = 10$, $C(x) = x^{2.3}$, and $r_A = r_B = 1$. The left panel shows the fixed-point mapping $x \mapsto \hat{x}(x, \delta(x))$ for the asymmetric setting with $q_A = 3/4$; the right panel shows the fixed-point mapping $x \mapsto \hat{x}(x, 0)$ for the symmetric setting in which $q_A = 1/2$.

The asymptotic inefficiency of the low-information equilibrium can be best understood via the equilibrium indifference condition (2), and when considering *regular* cost functions for which $\lim_{x\to 0} \frac{C'(x)x}{C(x)} \in (0, \infty)$.

As $x \to 0$ in all equilibrium sequences, indifference between (x, A, B) and $(0, A, A)$ requires asymptotic indifference between $(0, A, B)$ and $(0, A, A)$ and thus between $(0, A, A)$ and $(0, B, B)$; hence,

$$\frac{q_A r_A}{q_B r_B} \frac{\Pr(\text{piv}|z_A; \alpha_n)}{\Pr(\text{piv}|z_B; \alpha_n)} \to 1,$$

which we rewrite as

$$\begin{aligned}
\frac{q_B r_B}{q_A r_A} &= \lim_{n\to\infty}\left(\frac{\frac{1}{4}-MV(z_A;x,\delta)^2}{\frac{1}{4}-MV(z_B;x,\delta)^2}\right)^n \\
&= \lim_{n\to\infty}\left(1-\frac{MV(z_A;x,\delta)^2-MV(z_B;x,\delta)^2}{\frac{1}{4}-MV(z_B;x,\delta)^2}\right)^n \\
&= \lim_{n\to\infty} e^{-8n(\delta x)}. \qquad (8)
\end{aligned}$$

The last step used that $\lim_{n\to\infty}(1-\frac{y}{n})^n = e^{-y}$, that $MV(z_A;x,\delta)^2 - MV(z_B;x,\delta)^2 = (1-\delta)2\delta x$ and that the regularity of the cost function implies $\delta \to 0$ in all equilibrium sequences.[4]

The equilibrium parameters δ and x play fully symmetric roles in (8). If a sequence of δ_n and x_n solves it, then the sequence with reversed roles, i.e. with $\delta'_n = x_n$ and $x'_n = \delta_n$, also solves it.

Whether the vote share in z_B exceeds $\frac{1}{2}$, however, depends only on the comparison

$$\delta > 2x$$

given (5) and given that $\delta \to 0$. Consequently, when there is an efficient equilibrium sequence with $\delta_n < 2x_n$, this heuristic suggests why there is another one in which $\delta'_n > 2x'_n$ and thus outcomes are inefficient since A is elected in z_B with nonvanishing probability.

Remarks. Several remarks are worth making. First, the coordination problem is present even in small electorates. The example in Figure 2 shows existence for an electorate of just 21 voters. Second, the inefficient equilibrium sequence exists even when the information costs vanish arbitrarily fast to zero as $x \to 0$. Costs may even be exponentially low and, for example, given by $C(x) = \exp(-1/x)$ for $x \in (0, \frac{1}{2}]$ and $C(0) = 0$.

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[4] With regular cost functions, any equilibrium with information acquisition requires the election to have zero margins of victory as $n \to \infty$. Otherwise, there are not sufficient incentives to sustain the information acquisition. Zero margins, however, require that δ vanishes. Regularity excludes cost functions such as

$$C(x) = \begin{cases} \exp(-1/x), & x \in (0, \frac{1}{2}], \\ 0, & x = 0. \end{cases}$$

where costs vanish faster than any power of x as $x \to 0$. Regularity is not required for Theorem 1.

A Basic Notation

Fix a symmetric strategy α used by the other $2n$ voters and write

$$\rho_A = q_A r_A, \qquad \rho_B = q_B r_B,$$

so that $\rho_A > \rho_B$, and

$$P_A = \Pr(\text{piv} \mid z_A; \alpha), \qquad P_B = \Pr(\text{piv} \mid z_B; \alpha),$$

and

$$R_A = P_A q_A r_A, \qquad R_B = P_B q_B r_B.$$

For a pair (x, δ), define

$$a_A(x, \delta) = \frac{\delta}{2} + (1 - \delta)x, \qquad a_B(x, \delta) = \frac{\delta}{2} - (1 - \delta)x. \tag{9}$$

Then

$$\pi(z_A; \alpha(x, \delta)) = \frac{1}{2} + a_A(x, \delta), \qquad \pi(z_B; \alpha(x, \delta)) = \frac{1}{2} + a_B(x, \delta).$$

B Proof of Lemma 1

We prove the lemma in three steps. First, we show that, for all sufficiently large n, every equilibrium with information acquisition has support

$$\{(x, A, B), (0, A, A)\}$$

for some $x \in (0, \frac{1}{2})$, so it is a strategy $\alpha(x, \delta)$ for some $(x, \delta) \in (0, \frac{1}{2}) \times (0, 1)$. Second, we show that every such equilibrium satisfies the two conditions in Lemma 1, namely (2) and (3). Third, we prove the converse: if $(x, \delta) \in (0, \frac{1}{2}) \times (0, 1)$ is such that $\alpha(x, \delta)$ satisfies (2) and (3), then $\alpha(x, \delta)$ is an equilibrium with information acquisition.

Step 1: The support of equilibria with information acquisition. Fix a voter and a symmetric strategy α used by the other $2n$ voters. Only pivotal events matter for payoff comparisons. Up to terms that are constant across the fixed voter's strategies, the payoff from choosing (x_i, A, B) is

$$V_I(x_i; \alpha) = (R_A + R_B)\left(\frac{1}{2} + x_i\right) - C(x_i).$$

The payoffs from the two uninformative pure-voting strategies are

$$V_A(\alpha) = R_A, \qquad V_B(\alpha) = R_B.$$

For any $x_i > 0$, the strategies (x_i, A, A) and (x_i, B, B) are strictly dominated by $(0, A, A)$ and $(0, B, B)$, respectively, because they induce the same vote but add a positive information cost. The strategy (x_i, B, A) is strictly dominated by (x_i, A, B) because the signal is informative and voting with the signal raises the probability of choosing the correct policy in each state. Hence any pure strategy with information acquisition that is used with positive probability in equilibrium must be of the form (x_i, A, B).

The function $V_I(\cdot; \alpha)$ is strictly concave because C is strictly convex. Moreover, in any best response with information acquisition, $R_A + R_B > 0$, and therefore

$$V_I'(0; \alpha) = R_A + R_B - C'(0) > 0.$$

Thus the best response uses at most one informative precision. For large enough n, this precision is interior. Indeed, for any fixed $\eta \in (0, 1/2)$, the pivotal likelihood is uniformly bounded above by

$$\binom{2n}{n} 4^{-n},$$

which converges to zero. Hence, uniformly over α,

$$R_A + R_B \leq (q_A r_A + q_B r_B)\binom{2n}{n} 4^{-n} < C'(\eta)$$

for all sufficiently large n. Strict concavity then rules out any maximizer $x_i \geq \eta$. Since $V_I'(0; \alpha) > 0$, the informative maximizer is positive. Thus, for all sufficiently large n, the optimal informative precision $x^*(\alpha)$ lies in $(0, 1/2)$ and satisfies

$$R_A + R_B = C'(x^*(\alpha)).$$

Since the payoff from $(0, A, B)$ is a convex combination of $V_A(\alpha)$ and $V_B(\alpha)$, and the same is true of $(0, B, A)$, neither signal-contingent strategy with zero precision can be strictly better than both uninformative pure-voting strategies. If

$V_A(\alpha) = V_B(\alpha)$, then $V_I(0;\alpha) = V_A(\alpha) = V_B(\alpha)$ and $V_I'(0;\alpha) > 0$, so the maximized informative payoff is strictly higher than both uninformative payoffs. If $V_A(\alpha) \neq V_B(\alpha)$, only the better uninformative pure-voting strategy can possibly be tied with the informative strategy. Hence a best response with information acquisition can have only one of the following supports:

$$\{(x^*, A, B)\}, \qquad \{(x^*, A, B), (0, A, A)\}, \qquad \{(x^*, A, B), (0, B, B)\}.$$

We now rule out the first and third possibilities for all sufficiently large n when $q_A r_A > q_B r_B$.

First consider the candidate equilibrium in which all voters use (x, A, B). Then $P_A = P_B = P$ and the interior first-order condition is

$$P(q_A r_A + q_B r_B) = C'(x).$$

The informative strategy must also be at least as good as always voting for A:

$$P(q_A r_A + q_B r_B)\left(\frac{1}{2} + x\right) - C(x) \geq P q_A r_A.$$

Using the first-order condition to substitute for $P(q_A r_A + q_B r_B)$ gives

$$C'(x)\left(\frac{1}{2} + x\right) - C(x) \geq C'(x)\frac{q_A r_A}{q_A r_A + q_B r_B}.$$

Since $x > 0$ and $C'(x) > 0$, division by $C'(x)$ yields

$$\frac{1}{2} + x - \frac{C(x)}{C'(x)} \geq \frac{q_A r_A}{q_A r_A + q_B r_B}.$$

Subtracting $\frac{1}{2}$ from both sides gives

$$x - \frac{C(x)}{C'(x)} \geq \frac{q_A r_A}{q_A r_A + q_B r_B} - \frac{1}{2}.$$

The right-hand side simplifies as

$$\frac{q_A r_A}{q_A r_A + q_B r_B} - \frac{1}{2} = \frac{2q_A r_A - (q_A r_A + q_B r_B)}{2(q_A r_A + q_B r_B)} = \frac{1}{2}\frac{q_A r_A - q_B r_B}{q_A r_A + q_B r_B}.$$

Rearranging thus gives

$$x - \frac{1}{2}\frac{q_A r_A - q_B r_B}{q_A r_A + q_B r_B} \geq \frac{C(x)}{C'(x)} \geq 0.$$

Thus x must be bounded away from zero. But the first-order condition and the uniform convergence of pivotal probabilities to zero imply $C'(x) \to 0$, hence $x \to 0$, along any sequence of pure informative equilibria. This contradiction rules out the pure informative case for all sufficiently large n.

Next consider a candidate equilibrium with support $\{(x, A, B), (0, B, B)\}$. Let $\beta \in (0, 1)$ be the probability assigned to $(0, B, B)$. Then

$$\pi(z_A; \alpha) = (1-\beta)\left(\frac{1}{2} + x\right), \qquad \pi(z_B; \alpha) = (1-\beta)\left(\frac{1}{2} - x\right).$$

Set

$$a = (1-\beta)\left(\frac{1}{2} + x\right), \qquad b = (1-\beta)\left(\frac{1}{2} - x\right).$$

Then $\pi(z_A; \alpha) = a$ and $\pi(z_B; \alpha) = b$, so

$$\begin{aligned}\pi(z_A; \alpha)(1 - \pi(z_A; \alpha)) - \pi(z_B; \alpha)(1 - \pi(z_B; \alpha)) &= a(1-a) - b(1-b)\\ &= (a-b)(1-a-b).\end{aligned}$$

Since

$$a - b = 2(1-\beta)x \qquad \text{and} \qquad 1 - a - b = 1 - (1-\beta) = \beta,$$

we obtain

$$\pi(z_A; \alpha)(1 - \pi(z_A; \alpha)) - \pi(z_B; \alpha)(1 - \pi(z_B; \alpha)) = 2\beta(1-\beta)x > 0.$$

Hence $P_A > P_B$, given (1). Together with $q_A r_A > q_B r_B$, this gives

$$V_A(\alpha) = P_A q_A r_A > P_B q_B r_B = V_B(\alpha).$$

Thus $(0, B, B)$ cannot be in the support of a best response. By way of exclusion, the support of any equilibrium with information acquisition is therefore

$$\{(x, A, B), (0, A, A)\}.$$

Step 2: Necessity of the two equilibrium conditions. Consider any equilibrium with information acquisition. By the first step, when n is sufficiently large, the equilibrium is a strategy $\alpha(x,\delta)$ with $(x,\delta) \in (0,\frac{1}{2}) \times (0,1)$ and support $\{(x,A,B),(0,A,A)\}$. Since x is interior and the strategy (x,A,B) is a best response, it must satisfy the first-order condition

$$\Pr(\text{piv} \mid z_A; \alpha(x,\delta))q_A r_A + \Pr(\text{piv} \mid z_B; \alpha(x,\delta))q_B r_B = C'(x),$$

which is precisely (3). Since both (x,A,B) and $(0,A,A)$ are used with positive probability, they must yield the same payoff. The indifference condition is precisely (2).

Step 3: Sufficiency. It remains to prove the converse. Suppose that $(x,\delta) \in (0,1/2) \times (0,1)$ is so that the strategy $\alpha(x,\delta)$ satisfies (2) and (3). The strategy $\alpha(x,\delta)$ has support $\{(x,A,B),(0,A,A)\}$, and (2) states that these two strategies yield the same payoffs. In the following, we argue that all strategies outside the support have weakly lower payoffs.

First, we show that $(0,B,B)$ is strictly worse. By (3),

$$R_A + R_B = C'(x).$$

By the indifference condition (2),

$$R_A = (R_A + R_B)\left(\frac{1}{2} + x\right) - C(x).$$

Substituting $R_A + R_B = C'(x)$ gives

$$R_A = C'(x)\left(\frac{1}{2} + x\right) - C(x).$$

Therefore,

$$R_B = C'(x) - R_A = C'(x) - C'(x)\left(\frac{1}{2} + x\right) + C(x) = C'(x)\left(\frac{1}{2} - x\right) + C(x).$$

Hence

$$\begin{aligned} V_A(\alpha(x,\delta)) - V_B(\alpha(x,\delta)) &= R_A - R_B \\ &= \left[C'(x)\left(\frac{1}{2} + x\right) - C(x)\right] - \left[C'(x)\left(\frac{1}{2} - x\right) + C(x)\right] \\ &= 2\big(xC'(x) - C(x)\big) > 0. \end{aligned}$$

The strict inequality follows from strict convexity of C and $C(0)=0$: for $x>0$,

$$C(x)=\int_0^x C'(t)\,dt < xC'(x).$$

Thus $(0,B,B)$ is strictly worse than the two strategies in the support.

Second, the uninformative signal-contingent strategies $(0,A,B)$ and $(0,B,A)$ are strictly worse because their payoffs are convex combinations of the payoffs R_A and R_B from $(0,A,A)$ and $(0,B,B)$; they are given by

$$\frac{1}{2}(R_A+R_B).$$

Third, as argued above, any costly strategy other than voting with the signal is dominated. Fourth, the payoff $V_I(y;\alpha(x,\delta))$ of any strategy (y,A,B) with $y\neq x$ yields a weakly lower payoff than (x,A,B), because under $\alpha(x,\delta)$, the payoff $V_I(\cdot;\alpha(x,\delta))$ is strictly concave, and (3) makes x its unique maximizer.

We conclude that every strategy in the support of $\alpha(x,\delta)$ is a best response. Hence $\alpha(x,\delta)$ is a symmetric equilibrium with information acquisition.

C Proof of Proposition 1

For a strategy $\alpha(x,\delta)$, the marginal benefit of a higher precision is

$$\mathrm{MB}_n(x,\delta)=\binom{2n}{n}\left[\rho_A\left(\frac{1}{4}-a_A(x,\delta)^2\right)^n+\rho_B\left(\frac{1}{4}-a_B(x,\delta)^2\right)^n\right].$$

Since

$$\frac{\partial a_A(x,\delta)}{\partial x}=1-\delta,\qquad \frac{\partial a_B(x,\delta)}{\partial x}=-(1-\delta),$$

differentiating MB_n with respect to x gives

$$\begin{aligned}\frac{\partial \mathrm{MB}_n(x,\delta)}{\partial x}=2n(1-\delta)\binom{2n}{n}\Bigg[&\rho_B a_B(x,\delta)\left(\frac{1}{4}-a_B(x,\delta)^2\right)^{n-1}\\ &-\rho_A a_A(x,\delta)\left(\frac{1}{4}-a_A(x,\delta)^2\right)^{n-1}\Bigg].\end{aligned}$$

The factor outside the square brackets is strictly positive. Hence the sign of $\partial\mathrm{MB}_n(x,\delta)/\partial x$ is the sign of the expression inside the square brackets.

If $\delta=0$, then $a_A(x,0)=x$ and $a_B(x,0)=-x$. The expression inside the square brackets becomes

$$-(\rho_A + \rho_B)x\left(\frac{1}{4} - x^2\right)^{n-1} < 0$$

for every $x \in (0, 1/2)$. Thus $\mathrm{MB}_n(\cdot, 0)$ is strictly decreasing. In the remainder of the proof, fix $\delta > 0$.

Claim 1 *If $a_B(x,\delta) \leq 0$, then*

$$\frac{\partial \mathrm{MB}_n(x,\delta)}{\partial x} < 0.$$

If $a_B(x,\delta) > 0$ and

$$s = \frac{2(1-\delta)x}{\delta},$$

then the sign of $\partial\mathrm{MB}_n(x,\delta)/\partial x$ is the sign of $\Lambda_n(s,\delta)$, where

$$\Lambda_n(s,\delta) = \log\frac{\rho_B}{\rho_A} + \log\frac{1-s}{1+s} + (n-1)\log\frac{1-\delta^2(1-s)^2}{1-\delta^2(1+s)^2}.$$

Proof. If $a_B(x,\delta) \leq 0$, then the first term inside the square brackets in the derivative of MB_n is weakly negative, while the second term is strictly negative since $a_A(x,\delta) > 0$. Hence the overall derivative is strictly negative.

Now suppose $a_B(x,\delta) > 0$. Then $s < 1$, and

$$a_A(x,\delta) = \frac{\delta}{2}(1+s), \qquad a_B(x,\delta) = \frac{\delta}{2}(1-s).$$

The derivative $\partial_x\mathrm{MB}_n(x,\delta)$ is positive if and only if

$$\rho_B a_B(x,\delta)\left(\frac{1}{4} - a_B(x,\delta)^2\right)^{n-1} > \rho_A a_A(x,\delta)\left(\frac{1}{4} - a_A(x,\delta)^2\right)^{n-1}.$$

Taking logs, the inequality is equivalent to

$$\log\frac{\rho_B}{\rho_A} + \log\frac{a_B(x,\delta)}{a_A(x,\delta)} + (n-1)\log\frac{\frac{1}{4} - a_B(x,\delta)^2}{\frac{1}{4} - a_A(x,\delta)^2} > 0.$$

($x < 1/2$ implies $a_A(x,\delta) < 1/2$, so all logarithms below are well defined.) Using

$$\frac{a_B(x,\delta)}{a_A(x,\delta)} = \frac{1-s}{1+s},$$

and

$$\frac{\frac{1}{4}-a_B(x,\delta)^2}{\frac{1}{4}-a_A(x,\delta)^2}=\frac{\frac{1}{4}-\frac{\delta^2}{4}(1-s)^2}{\frac{1}{4}-\frac{\delta^2}{4}(1+s)^2}=\frac{1-\delta^2(1-s)^2}{1-\delta^2(1+s)^2},$$

the preceding inequality becomes exactly $\Lambda_n(s,\delta)>0$. ■

Claim 2 *If $a_B(x,\delta)>0$, the sign of $\partial\Lambda_n/\partial t$ is the sign of*

$$Q_n(p)=-(2n-1)p^2+\frac{n-2}{2}p+\frac{\delta^2}{4}-\frac{1}{16},$$

where

$$t=(1-\delta)x, \qquad p=a_A(x,\delta)a_B(x,\delta)=\frac{\delta^2}{4}-(1-\delta)^2x^2.$$

Moreover, for all sufficiently large n:

- *if $0<\delta<1/2$, then $\Lambda_n(\cdot,\delta)$ has at most one local maximum;*
- *if $1/2<\delta<1$, then $\Lambda_n(\cdot,\delta)$ is strictly increasing for $x\in(0,1/2)$.*

Proof. Throughout the proof, use

$$m=\frac{\delta}{2}, \qquad a=m+t=a_A(x,\delta), \qquad b=m-t=a_B(x,\delta).$$

Since $s=t/m$ and $m>0$, differentiating with respect to s or t gives the same sign. It is convenient to differentiate with respect to t. Since $a=m+t$ and $b=m-t$, we have $da/dt=1$ and $db/dt=-1$. Using the log representation

$$\Lambda_n=\log\frac{\rho_B}{\rho_A}+\log\frac{b}{a}+(n-1)\log\frac{\frac{1}{4}-b^2}{\frac{1}{4}-a^2},$$

we obtain

$$\frac{\partial\Lambda_n}{\partial t}=-\left(\frac{1}{b}+\frac{1}{a}\right)+(n-1)\left[\frac{2b}{\frac{1}{4}-b^2}+\frac{2a}{\frac{1}{4}-a^2}\right].$$

Equivalently,

$$\frac{\partial\Lambda_n}{\partial t}=2(n-1)\left[\frac{b}{\frac{1}{4}-b^2}+\frac{a}{\frac{1}{4}-a^2}\right]-\left(\frac{1}{b}+\frac{1}{a}\right).$$

We now simplify the two terms. First,

$$\begin{aligned}\frac{b}{\frac{1}{4}-b^2}+\frac{a}{\frac{1}{4}-a^2} &= \frac{b(\frac{1}{4}-a^2)+a(\frac{1}{4}-b^2)}{(\frac{1}{4}-a^2)(\frac{1}{4}-b^2)}\\ &= \frac{\frac{1}{4}(a+b)-ab(a+b)}{(\frac{1}{4}-a^2)(\frac{1}{4}-b^2)}\\ &= \frac{\delta(\frac{1}{4}-p)}{(\frac{1}{4}-a^2)(\frac{1}{4}-b^2)}.\end{aligned}$$

Second,

$$\frac{1}{b}+\frac{1}{a}=\frac{a+b}{ab}=\frac{\delta}{p}.$$

Since $\delta > 0$, $p > 0$, and $(\frac{1}{4}-a^2)(\frac{1}{4}-b^2) > 0$, multiplication of $\frac{\partial \Lambda_n}{\partial t}$ by

$$\frac{p}{\delta}\left(\frac{1}{4}-a^2\right)\left(\frac{1}{4}-b^2\right)$$

shows that the sign of $\partial\Lambda_n/\partial t$ is the sign of

$$2(n-1)p\left(\frac{1}{4}-p\right)-\left(\frac{1}{4}-a^2\right)\left(\frac{1}{4}-b^2\right).$$

It remains to express the last product in terms of p. We have

$$\left(\frac{1}{4}-a^2\right)\left(\frac{1}{4}-b^2\right)=\frac{1}{16}-\frac{1}{4}(a^2+b^2)+a^2b^2.$$

Since

$$a^2+b^2=(a+b)^2-2ab=\delta^2-2p,$$

and $a^2b^2=p^2$, this becomes

$$\left(\frac{1}{4}-a^2\right)\left(\frac{1}{4}-b^2\right)=\frac{1}{16}-\frac{\delta^2}{4}+\frac{p}{2}+p^2.$$

Therefore the sign of $\partial\Lambda_n/\partial t$ is the sign of

$$2(n-1)p\left(\frac{1}{4}-p\right)-\left(\frac{1}{16}-\frac{\delta^2}{4}+\frac{p}{2}+p^2\right).$$

Expanding,

$$2(n-1)p\left(\frac{1}{4}-p\right)=\frac{n-1}{2}p-2(n-1)p^2.$$

Thus the preceding expression equals

$$-(2n-1)p^2 + \frac{n-2}{2}p + \frac{\delta^2}{4} - \frac{1}{16} = Q_n(p),$$

as claimed.

Now suppose $0 < \delta < 1/2$. If $a_B(x, \delta) > 0$, then

$$0 < x < \frac{\delta}{2(1-\delta)}.$$

Hence

$$0 < (1-\delta)^2 x^2 < \frac{\delta^2}{4}, \qquad \text{so} \qquad p = \frac{\delta^2}{4} - (1-\delta)^2 x^2 \in \left(0, \frac{\delta^2}{4}\right).$$

Since $\delta < 1/2$, this implies $p \in (0, 1/16)$. Differentiating Q_n gives

$$Q_n'(p) = -2(2n-1)p + \frac{n-2}{2}.$$

Since $p < 1/16$,

$$Q_n'(p) > -\frac{2n-1}{8} + \frac{n-2}{2} = \frac{2n-7}{8} > 0$$

for all sufficiently large n. Hence Q_n is increasing in p. Since

$$p = \frac{\delta^2}{4} - (1-\delta)^2 x^2$$

is decreasing in x, and hence in t, the derivative $\partial \Lambda_n / \partial t$ changes sign at most once, from positive to negative. Therefore $\Lambda_n(\cdot, \delta)$ is either decreasing throughout, or else increasing and then decreasing. In particular, it has at most one local maximum.

Now suppose $1/2 \le \delta < 1$. For every $x \in (0, 1/2)$,

$$a_B(x, \delta) = \frac{\delta}{2} - (1-\delta)x > \frac{\delta}{2} - \frac{1-\delta}{2} = \frac{2\delta - 1}{2} \ge 0.$$

Thus $a_B(x, \delta) > 0$ for all $x \in (0, 1/2)$. The variable p ranges from

$$p_0 = \frac{\delta^2}{4} \qquad \text{as } x \downarrow 0$$

down to

$$p_1 = \frac{2\delta - 1}{4} \qquad \text{as } x \uparrow \frac{1}{2}.$$

The quadratic Q_n is concave in p, so its minimum on $[p_1, p_0]$ is attained at one of the endpoints. Substituting $p_0 = \delta^2/4$ gives

$$\begin{aligned} Q_n(p_0) &= -(2n-1)\frac{\delta^4}{16} + \frac{n-2}{2}\frac{\delta^2}{4} + \frac{\delta^2}{4} - \frac{1}{16} \\ &= \frac{-(2n-1)\delta^4 + 2n\delta^2 - 1}{16}. \end{aligned}$$

Since $\delta < 1$, we see that $Q_n(p_0) > 0$ for all sufficiently large n. Similarly, substituting $p_1 = (2\delta - 1)/4$ gives[5]

$$Q_n(p_1) = -(2n-1)\frac{(2\delta-1)^2}{16} + \frac{n-2}{2}\frac{2\delta-1}{4} + \frac{\delta^2}{4} - \frac{1}{16}$$

Since $\frac{1}{2} < \delta < 1$, we see that $Q_n(p_1) > 0$ for n sufficiently large. Concavity implies

$$Q_n(p) > 0$$

for every $x \in (0, 1/2)$. Therefore $\partial\Lambda_n/\partial t > 0$, so $\Lambda_n(\cdot, \delta)$ is strictly increasing for $x \in (0, 1/2)$. ■

Claim 3 *For all sufficiently large n, there exists $\delta_{1,n} \in (0, 1/2)$ such that:*

$$0 < \delta < \delta_{1,n} \quad \Longrightarrow \quad \mathrm{MB}_n(\cdot, \delta) \text{ is strictly decreasing for } x \in (0, 1/2),$$

and

$$\delta_{1,n} < \delta < 1/2 \quad \Longrightarrow \quad \mathrm{MB}_n(\cdot, \delta) \text{ is strictly decreasing, strictly increasing, then strictly decreasing.}$$

Proof. For $0 < \delta < 1/2$, define

$$L_n(\delta) = \sup_{s \in (0,1)} \Lambda_n(s, \delta),$$

and set

$$L_n(0) = \log \frac{\rho_B}{\rho_A}.$$

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[5] For the last equality, set $z = 2\delta - 1$. Then $4\delta^2 - 1 = (z+1)^2 - 1 = z^2 + 2z$. After putting all terms over the common denominator 16, the numerator is

$$\begin{aligned} -(2n-1)z^2 + 2(n-2)z + 4\delta^2 - 1 &= -(2n-1)z^2 + 2(n-2)z + z^2 + 2z \\ &= 2(n-1)z(1-z). \end{aligned}$$

Since $1 - z = 2(1 - \delta)$, this numerator equals $4(1-\delta)(2\delta-1)(n-1)$. Dividing by 16 gives the displayed expression.

We make several observations in order to define $\delta_{1,n}$: First, L_n is weakly increasing. This is because, for each fixed $s \in (0,1)$, the function $\Lambda_n(s,\delta)$ is increasing in δ. Indeed, only the last term in Λ_n depends on δ, and

$$\begin{aligned}
\frac{\partial}{\partial\delta} \log \frac{1-\delta^2(1-s)^2}{1-\delta^2(1+s)^2} &= -\frac{2\delta(1-s)^2}{1-\delta^2(1-s)^2} + \frac{2\delta(1+s)^2}{1-\delta^2(1+s)^2} \\
&= 2\delta \left[\frac{(1+s)^2}{1-\delta^2(1+s)^2} - \frac{(1-s)^2}{1-\delta^2(1-s)^2} \right] > 0.
\end{aligned}$$

Second, $L_n(0) = \log \frac{\rho_B}{\rho_A} < 0$. Third, for every fixed $\bar{\delta} \in (0,1/2)$, $L_n(\bar{\delta}) > 0$ if n is sufficiently large. This is because, for every fixed $s \in (0,1)$,

$$\log \frac{1-\bar{\delta}^2(1-s)^2}{1-\bar{\delta}^2(1+s)^2} > 0,$$

so the last term in $\Lambda_n(s,\bar{\delta})$ grows linearly in n, implying the claim.

We conclude that, for all sufficiently large n, the threshold

$$\delta_{1,n} = \inf\{\delta \in [0,1/2) : L_n(\delta) \geq 0\}$$

is well defined and belongs to $(0,1/2)$.

If $0 < \delta < \delta_{1,n}$, then $\Lambda_n(s,\delta) < 0$ for every $s \in (0,1)$. If $a_B(x,\delta) > 0$, the first claim implies $\partial_x \mathrm{MB}_n(x,\delta) < 0$. If $a_B(x,\delta) \leq 0$, the first claim also implies $\partial_x \mathrm{MB}_n(x,\delta) < 0$. Therefore $\mathrm{MB}_n(\cdot,\delta)$ is strictly decreasing for $x \in (0,1/2)$.

Consider now $\delta_{1,n} < \delta < 1/2$. By the second claim, $\Lambda_n(\cdot,\delta)$ has at most one local maximum. Since

$$\lim_{s \downarrow 0} \Lambda_n(s,\delta) = \log \frac{\rho_B}{\rho_A} < 0, \qquad \lim_{s \uparrow 1} \Lambda_n(s,\delta) = -\infty,$$

this maximum is taken at some interior $s \in (0,1)$. Thus, since Λ_n is strictly increasing in δ for any fixed s, L_n is *strictly* increasing in δ. Consequently, $L_n(\delta) > 0$ for $\delta > \delta_{1,n}$. We conclude that Λ_n crosses zero exactly twice in $s \in (0,1)$. By the first claim, the sign of $\partial_x \mathrm{MB}_n(x,\delta)$ is negative, then positive, then negative if $a_B(x,\delta) > 0$, i.e. for

$$x \in \left(0, \frac{\delta}{2(1-\delta)}\right).$$

For

$$x \in \left[\frac{\delta}{2(1-\delta)}, \frac{1}{2}\right),$$

we have $a_B(x, \delta) \leq 0$, so the first claim implies $\partial_x \mathrm{MB}_n(x, \delta) < 0$. We conclude that $\mathrm{MB}_n(\cdot, \delta)$ is strictly decreasing, then strictly increasing, and then strictly decreasing for $x \in (0, 1/2)$. ■

Claim 4 *If* $1/2 < \delta < 1$, *then* $\mathrm{MB}_n(\cdot, \delta)$ *is strictly decreasing and then strictly increasing for* $x \in (0, 1/2)$.

Proof. Fix $1/2 < \delta < 1$. As shown above,

$$a_B(x, \delta) > 0 \qquad \text{for every } x \in (0, 1/2).$$

Hence, by the first claim, the sign of $\partial_x \mathrm{MB}_n(x, \delta)$ is the sign of $\Lambda_n(s, \delta)$, where

$$s = \frac{2(1-\delta)x}{\delta}.$$

For $x \in (0, \frac{1}{2})$, this variable satisfies

$$s \in (0, \frac{1-\delta}{\delta}).$$

By the second claim, $\Lambda_n(\cdot, \delta)$ is strictly increasing. Also,

$$\lim_{s \downarrow 0} \Lambda_n(s, \delta) = \log \frac{\rho_B}{\rho_A} < 0.$$

At the other endpoint, as $s \uparrow \frac{(1-\delta)}{\delta}$,

$$1 - \delta^2(1+s)^2 \downarrow 0.$$

Therefore the last logarithmic term in $\Lambda_n(s, \delta)$ diverges to $+\infty$. If $\delta > 1/2$, the second term $\log \frac{1-s}{1+s}$ remains finite at the upper endpoint. Hence, for all sufficiently large n,

$$\lim_{s \uparrow (1-\delta)/\delta} \Lambda_n(s, \delta) = +\infty.$$

Thus $\Lambda_n(\cdot, \delta)$ is strictly increasing, starts below zero, and tends to $+\infty$. It crosses zero exactly once. Therefore $\partial_x \mathrm{MB}_n(x, \delta)$ is negative before this crossing and positive after it. Hence $\mathrm{MB}_n(\cdot, \delta)$ is strictly decreasing and then strictly increasing for $x \in (0, 1/2)$. ■

Combining the preliminary case $\delta = 0$ with Claims 3 and 4 proves the proposition.

D Mathematical Preliminaries for the Proof of Theorem 1

Central limit theorem. Consider a sequence of strategies $(\alpha_n)_{n\in\mathbb{N}}$. Define the standardized distance

$$\Delta_n(z;\alpha_n) = \frac{\pi(z;\alpha_n) - \frac{n}{2n+1}}{s_n(z;\alpha_n)}, \qquad s_n(z;\alpha_n) = \left(\frac{2n+1}{\pi(z;\alpha_n)(1-\pi(z;\alpha_n))}\right)^{-1/2}. \tag{10}$$

The term $s_n(z;\alpha_n)$ is the standard deviation of the vote share for A in state z. Thus $\Delta_n(z;\alpha_n)$ measures the distance between the expected vote share for A and the majority threshold, in standard deviations.

The central limit theorem implies that, whenever $\Delta_n(z;\alpha_n)$ converges in $\mathbb{R}\cup\{\infty,-\infty\}$,

$$\lim_{n\to\infty} \Pr(A \text{ is elected} \mid z;\alpha_n) = \lim_{n\to\infty} \Phi\big(\Delta_n(z;\alpha_n)\big), \tag{11}$$

where Φ is the standard normal distribution function.

Stirling approximation. We will also repeatedly use the Stirling approximation of the pivotal likelihood in (1):

$$\Pr(\text{piv} \mid z;\alpha_n) = \frac{1+o(1)}{\sqrt{\pi n}}\Big(4\pi_n(z;\alpha_n)(1-\pi_n(z;\alpha_n))\Big)^n. \tag{12}$$

Thus, the pivotal likelihood is exponentially small unless $\pi_n(z;\alpha_n)$ is within order $n^{-1/2}$ of $1/2$. Based on the Stirling approximation, one obtains

$$\begin{aligned}\Pr(\text{piv} \mid z_A;\alpha_n) = &\quad \frac{1+o(1)}{\sqrt{\pi n}}\Big(4(\frac{1}{2}+a_{A,n})(\frac{1}{2}-a_{A,n})\Big)^n \\ = &\quad \frac{1+o(1)}{\sqrt{\pi n}}(1-4a_{A,n}^2)^n.\end{aligned}$$

where the second equality uses the differences of squares identity. Consequently,

$$\frac{\Pr(\text{piv} \mid z_A;\alpha)}{n^{-\frac{1}{2}}} \to \frac{1}{\sqrt{\pi}}e^{-y} \tag{13}$$

for

$$y = \lim_{n\to\infty} 4na_{A,n}^2 \in \mathbb{R}\cup\{\infty\}$$

since $\lim_{n\to\infty}(1-\frac{y}{n})^n = e^{-y}$ and since the exponential function is continuous.

Uniform Bound for Pivotal Likelihood. We repeatedly use a uniform bound for the pivotal likelihood. Namely,

$$\Pr(\text{piv} \mid z;\alpha) = \binom{2n}{n}\Big(\pi(z;\alpha)(1-\pi(z;\alpha))\Big)^n \leq \binom{2n}{n}4^{-n}. \tag{14}$$

This holds because the function $q(1-q)$ has its unique maximum at $q=\frac{1}{2}$.

A Generalized Poincaré–Miranda theorem The central part of the proof relies on the following fixed-point theorem from Ekmekci *et al.* (2025), which is a generalization of the Poincaré–Miranda theorem.

Lemma 2 *Let* $f,g:[0,1]^2\to[-1,1]$ *be continuous. Suppose that* $f(0,t)<0$ *and* $f(1,t)>0$ *for all* $t\in[0,1]$*. Suppose further that* $g(r,0)>0$ *whenever* $f(r,0)=0$*, and* $g(r,1)<0$ *whenever* $f(r,1)=0$*. Then there exists* $(r^*,t^*)\in(0,1)^2$ *such that* $f(r^*,t^*)=g(r^*,t^*)=0$*.*

The result also applies to any domain homeomorphic to the unit square.

E Proof of Theorem 1

The first part of the theorem is the existence result of Martinelli (2006); see Theorem 4 therein. Now, we first prove the third part and then the second part of the theorem.

A useful observation is that the assumption $C''(0)=0$ implies

$$C'(y)=o(y) \quad \text{and} \quad C(y)=o(y^2) \qquad \text{as } y\downarrow 0. \tag{15}$$

E.1 Third part of Theorem 1

Consider any sequence of equilibria $\alpha(x_n,\delta_n)$ with information acquisition. Write

$$a_{A,n}=a_A(x_n,\delta_n)=\frac{\delta_n}{2}+(1-\delta_n)x_n, \qquad a_{B,n}=a_B(x_n,\delta_n)=\frac{\delta_n}{2}-(1-\delta_n)x_n.$$

We first claim that

$$\sqrt{n}\,a_{A,n}\to\infty.$$

Suppose not. Then, along a subsequence, $a_{A,n} = O(n^{-1/2})$. Since both terms in

$$a_{A,n} = \frac{\delta_n}{2} + (1-\delta_n)x_n$$

are nonnegative, this implies $\delta_n = O(n^{-1/2})$ and $x_n = O(n^{-1/2})$. By the Stirling approximation, $P_A(x_n, \delta_n)$ is bounded below by a positive multiple of $n^{-1/2}$ along this subsequence. Hence

$$\mathrm{MB}_n(x_n, \delta_n) = R_A(x_n, \delta_n) + R_B(x_n, \delta_n) \geq cn^{-1/2}$$

for some $c > 0$. But the first-order condition gives

$$C'(x_n) = \mathrm{MB}_n(x_n, \delta_n),$$

whereas $x_n = O(n^{-1/2})$ and (15) imply

$$C'(x_n) \leq C'(Kn^{-1/2}) = o(n^{-1/2})$$

for some $K > 0$, a contradiction. Thus $\sqrt{n}\, a_{A,n} \to \infty$.

Next we claim that

$$\sqrt{n}\, |a_{B,n}| \to \infty.$$

Suppose not. Then, along a subsequence, $|a_{B,n}| = O(n^{-1/2})$. By the Stirling approximation, $P_B(x_n, \delta_n)$ is bounded below by a positive multiple of $n^{-1/2}$.

We now use the equilibrium indifference condition. Since

$$\mathrm{MB}_n(x_n, \delta_n) = R_A(x_n, \delta_n) + R_B(x_n, \delta_n),$$

the indifference condition can be written as

$$\big(R_A(x_n, \delta_n) + R_B(x_n, \delta_n)\big)\left(\frac{1}{2} + x_n\right) - C(x_n) = R_A(x_n, \delta_n).$$

Dividing by $R_A(x_n, \delta_n) + R_B(x_n, \delta_n)$ gives

$$\frac{R_A(x_n, \delta_n)}{R_A(x_n, \delta_n) + R_B(x_n, \delta_n)} = \frac{1}{2} + x_n - \frac{C(x_n)}{R_A(x_n, \delta_n) + R_B(x_n, \delta_n)}.$$

Using the first-order condition

$$R_A(x_n, \delta_n) + R_B(x_n, \delta_n) = C'(x_n),$$

we obtain

$$\frac{R_A(x_n,\delta_n)}{R_A(x_n,\delta_n)+R_B(x_n,\delta_n)}=\frac{1}{2}+x_n-\frac{C(x_n)}{C'(x_n)}.$$

By convexity and $C(0)=0$, we have $C(x_n)\le x_nC'(x_n)$, and hence the right-hand side is at least $1/2$. Therefore

$$R_A(x_n,\delta_n)\ge R_B(x_n,\delta_n).$$

Since $R_z=\rho_zP_z$, the lower bound on $P_B(x_n,\delta_n)$ implies that $P_A(x_n,\delta_n)$ is also bounded below by a positive multiple of $n^{-1/2}$. By the Stirling approximation, this implies $a_{A,n}=O(n^{-1/2})$, contradicting the first claim. Thus $\sqrt{n}\,|a_{B,n}|\to\infty$.

By the central limit theorem, $\sqrt{n}\,a_{A,n}\to\infty$ implies that A is elected in state z_A with probability converging to one. In z_B, every subsequence has a further subsequence along which either $a_{B,n}>0$ or $a_{B,n}<0$. In the first case, $\sqrt{n}\,a_{B,n}\to\infty$, so A is elected in z_B with probability converging to one. In the second case, $\sqrt{n}\,a_{B,n}\to-\infty$, so B is elected in z_B with probability converging to one. Therefore, if the original sequence has convergent outcome probabilities, only one of these two limiting outcome types can occur: either the sequence is asymptotically efficient, or A is elected with probability converging to one in both states.

E.2 Second part of Theorem 1

We prove the second part by constructing a sequence of equilibria α_n with information acquisition and asymptotically inefficient outcomes. Throughout, we use the notation $\hat{x}_n$ for the unique precision that maximizes the expected utility across all strategies (x,A,B), given a candidate strategy α_n. Equivalently, $\hat{x}_n$ solves the first-order condition

$$\mathrm{MB}_n(x_n,\delta_n)=C'(\hat{x}_n).$$

The proof proceeds in three steps. First, we define a domain D_n of candidate strategies and relate the two equilibrium conditions (2) and (3) in Lemma 1 to roots of two functions F_n and G_n. Second, we establish the boundary conditions of the generalized Poincaré–Miranda theorem (Lemma 2) applied to F_n and G_n on D_n. Third, we use the theorem to construct an equilibrium sequence in which A is elected with probability converging to one in both states.

Step 1: The Domain of Candidate Strategies and the Equilibrium Conditions. We consider candidate equilibrium strategies α_n, given by pairs $(x_n,\delta_n)\in$

D_n for the domain

$$D_n = \{(x, \delta) : x \in \left[0, Mn^{-\frac{1}{2}}\right], \delta \in \left[3Mn^{-\frac{1}{2}}, \overline{\delta}_n\right]\} \tag{16}$$

for some $M > 0$ and $\overline{\delta}_n$ close to 1 that will be defined in the course of the proof.

For $(x, \delta) \in D_n$, define

$$F_n(x, \delta) = \max_{\tilde{x} \in [0,1/2]} \left\{ \mathrm{MB}_n(x, \delta) \left(\frac{1}{2} + \tilde{x} \right) - C(\tilde{x}) - R_A(x, \delta) \right\},$$

and

$$G_n(x, \delta) = \mathrm{MB}_n(x, \delta) - C'(x).$$

At any interior root (x, δ) with $F_n(x, \delta) = G_n(x, \delta) = 0$, the equation $G_n(x, \delta) = 0$ is exactly the first-order condition (3). Since the maximand in the definition of F_n is strictly concave in $\tilde{x}$, the first-order condition (3) implies that $\tilde{x} = x$ is its unique maximizer. Hence $F_n(x, \delta) = 0$ becomes $\mathrm{MB}_n(x, \delta) \left(\frac{1}{2} + x\right) - C(x) - R_A(x, \delta) = 0$, which is precisely the indifference condition (2) between (x, A, B) and $(0, A, A)$. Thus the two equations $F_n = 0$ and $G_n = 0$ are exactly the two equilibrium conditions in Lemma 1.

Step 2: The boundary conditions of the generalized Poincaré–Miranda theorem.

Boundary $x = 0$**.** Let $x = 0$. Then $\pi(z_A; \alpha(0, \delta)) = \pi(z_B; \alpha(0, \delta))$ and $P_A = P_B =: P_n(\delta)$. Since

$$P_n(\delta) \leq \binom{2n}{n} 4^{-n} = O(n^{-1/2}),$$

the maximizer in the definition of $F_n(0, \delta)$ converges uniformly to 0 for all δ. Hence, uniformly,

$$F_n(0, \delta) \leq P_n(\delta) \left[-\frac{\rho_A - \rho_B}{2} + o(1) \right] < 0$$

for all sufficiently large n.

Intuitively, since all voters strictly prefer A given the prior belief, for n sufficiently large, $\hat{x}_n$ is sufficiently small so that after any signal $s \in \{s_A, s_B\}$, the voter strictly prefers A over B. Thus, $(\hat{x}_n, A, A)$ yields strictly more utility than $(\hat{x}_n, A, B)$. Further, $(0, A, A)$ yields strictly more utility than $(\hat{x}_n, A, A)$ since the voter does not acquire costly information. Thus, all voters strictly prefer $(0, A, A)$ to $(\hat{x}_n, A, B)$, as captured by the inequality $F_n(0, \delta) < 0$.

Boundary $x = Mn^{-\frac{1}{2}}$. Choose $M > 0$ so large that

$$\frac{\rho_A}{\rho_B} e^{-8M^2} < 1. \tag{17}$$

We show that this implies $F_n(Mn^{-1/2}, \delta) > 0$ for all $\delta \geq 3Mn^{-1/2}$ and all sufficiently large n. Let $x = Mn^{-1/2}$. Following the same algebra as for the Stirling approximation and since the common binomial factor cancels out,

$$\frac{P_A(x,\delta)}{P_B(x,\delta)} = \left(\frac{1 - 4a_A(x,\delta)^2}{1 - 4a_B(x,\delta)^2} \right)^n .$$

Since

$$a_A(x,\delta)^2 - a_B(x,\delta)^2 = 2\delta(1-\delta)x, \tag{18}$$

we can rewrite this as

$$\frac{P_A(x,\delta)}{P_B(x,\delta)} = \left(1 - \frac{a_A(x,\delta)^2 - a_B(x,\delta)^2}{\frac{1}{4} - a_B(x,\delta)^2} \right)^n = \left(1 - \frac{2\delta(1-\delta)x}{\frac{1}{4} - a_B(x,\delta)^2} \right)^n . \tag{19}$$

Moreover,

$$\frac{1}{4} - a_B(x,\delta)^2 = \left(\frac{1}{2} - a_B(x,\delta) \right) \left(\frac{1}{2} + a_B(x,\delta) \right).$$

Using (9), we rewrite

$$\frac{1}{2} - a_B(x,\delta) = (1-\delta) \left(\frac{1}{2} + x \right).$$

Since $\frac{1}{2} + a_B(x,\delta) \leq 1$, we obtain the bound

$$\frac{1}{4} - a_B(x,\delta)^2 \leq (1-\delta) \left(\frac{1}{2} + x \right).$$

Thus

$$\frac{P_A(x,\delta)}{P_B(x,\delta)} \leq \left(1 - \frac{2\delta x}{\frac{1}{2} + x} \right)^n . \tag{20}$$

For $x = Mn^{-1/2}$ and $\delta \geq 3Mn^{-1/2}$, we have $\frac{2\delta x}{\frac{1}{2}+x} \geq \frac{6M^2/n}{\frac{1}{2}+Mn^{-1/2}}$. Since, for all n sufficiently large,

$$\frac{1}{2} + Mn^{-1/2} \leq \frac{3}{4},$$

it holds

$$\frac{2\delta x}{\frac{1}{2}+x} \geq \frac{8M^2}{n},$$

so that the term on the right side of (20) is at most $(1-8M^2/n)^n$. Using $\lim_{n\to\infty}(1-\frac{y}{n})^n = e^{-y}$,

$$\frac{P_A(x,\delta)}{P_B(x,\delta)} \leq e^{-8M^2}.$$

By (17), this implies

$$R_A(x,\delta) < R_B(x,\delta).$$

Evaluating the maximand in the definition of F_n at $\tilde{x} = 0$ therefore gives

$$F_n(x,\delta) \geq \frac{R_A(x,\delta)+R_B(x,\delta)}{2} - R_A(x,\delta) = \frac{R_B(x,\delta)-R_A(x,\delta)}{2} > 0.$$

Therefore $F_n(x,\delta) > 0$ for $x = Mn^{-1/2}$.

Boundary $\delta = 3Mn^{-\frac{1}{2}}$. Let $\delta = 3Mn^{-1/2}$. It follows from (9) that, for $x \in [0, Mn^{-1/2}]$, the terms $4na_A(x,\delta)^2$ and $4na_B(x,\delta)^2$ are bounded above and below by positive constants, uniformly in x. Thus, (13) implies that $\mathrm{MB}_n(x,\delta)/n^{-1/2}$ is bounded below by a positive constant, uniformly in x. By (15) and convexity of C,

$$C'(x) \leq C'(Mn^{-1/2}) = o(n^{-1/2}),$$

and

$$G_n(x, 3Mn^{-1/2}) = \mathrm{MB}_n(x, 3Mn^{-1/2}) - C'(x) > 0$$

for all sufficiently large n.

Boundary $\delta = \overline{\delta}_n$. Choose $\kappa > 0$ so small that

$$\frac{\rho_B}{\rho_A} e^{17\kappa} < 1. \tag{21}$$

For each sufficiently large n, choose $\overline{\delta}_n \in (3Mn^{-1/2}, 1)$ sufficiently close to one such that $\overline{\delta}_n \geq 1/2$ and

$$\sup_{x\in[0,Mn^{-1/2}]} \mathrm{MB}_n(x, \overline{\delta}_n) < C'(\kappa/n). \tag{22}$$

Such a choice is possible because, for fixed n, the pivotal probabilities converge uniformly to zero on $[0, Mn^{-1/2}]$ as $\delta \to 1$.

We verify the required boundary condition. Let $x \in [0, Mn^{-1/2}]$ satisfy $F_n(x, \overline{\delta}_n) = 0$. We first claim that $x > 2\kappa/n$. Suppose instead that $x \leq 2\kappa/n$.

Applying steps analogous to the derivation of (19) gives

$$\frac{P_B(x, \overline{\delta}_n)}{P_A(x, \overline{\delta}_n)} = \left(1 + \frac{a_A(x, \overline{\delta}_n)^2 - a_B(x, \overline{\delta}_n)^2}{\frac{1}{4} - a_A(x, \overline{\delta}_n)^2}\right)^n.$$

Moreover,

$$\frac{1}{4} - a_A(x, \overline{\delta}_n)^2 = \left(\frac{1}{2} - a_A(x, \overline{\delta}_n)\right)\left(\frac{1}{2} + a_A(x, \overline{\delta}_n)\right),$$

and, by (9),

$$\frac{1}{2} - a_A(x, \overline{\delta}_n) = (1 - \overline{\delta}_n)\left(\frac{1}{2} - x\right).$$

Combining these observations with (18),

$$\frac{a_A(x, \overline{\delta}_n)^2 - a_B(x, \overline{\delta}_n)^2}{\frac{1}{4} - a_A(x, \overline{\delta}_n)^2} = \frac{2\overline{\delta}_n x}{\left(\frac{1}{2} - x\right)\left(\frac{1}{2} + a_A(x, \overline{\delta}_n)\right)}.$$

If $x \leq 2\kappa/n$, then the numerator is at most $4\kappa/n$. Moreover, since $a_A(x, \overline{\delta}_n) \geq 0$ and $x \to 0$, the denominator is bounded below by a number arbitrarily close to $1/4$. Hence, for all sufficiently large n, the last fraction is bounded above by $17\kappa/n$. Therefore

$$\frac{P_B(x, \overline{\delta}_n)}{P_A(x, \overline{\delta}_n)} \leq \left(1 + \frac{17\kappa}{n}\right)^n \leq e^{17\kappa}.$$

Hence, by (21), there exists $\eta < 1$, independent of n, such that, for all sufficiently large n,

$$R_B(x, \overline{\delta}_n) \leq \eta R_A(x, \overline{\delta}_n).$$

Let $\widehat{x}_n$ denote the maximizer in the definition of $F_n(x, \overline{\delta}_n)$. By (22), the objective in the definition of F_n has negative derivative at $\tilde{x} = \kappa/n$. Since this objective is strictly concave in $\tilde{x}$, its maximizer $\widehat{x}_n$ satisfies $\widehat{x}_n \leq \kappa/n$. Using $\mathrm{MB}_n = R_A + R_B$, we get

$$
\begin{aligned}
F_n(x,\overline{\delta}_n) &= \mathrm{MB}_n(x,\overline{\delta}_n)\left(\frac{1}{2}+\widehat{x}_n\right) - C(\widehat{x}_n) - R_A(x,\overline{\delta}_n) \\
&= -\frac{R_A(x,\overline{\delta}_n) - R_B(x,\overline{\delta}_n)}{2} + \mathrm{MB}_n(x,\overline{\delta}_n)\widehat{x}_n - C(\widehat{x}_n) \\
&\le -\frac{R_A(x,\overline{\delta}_n) - R_B(x,\overline{\delta}_n)}{2} + \mathrm{MB}_n(x,\overline{\delta}_n)\frac{\kappa}{n}.
\end{aligned}
$$

Since $R_B(x,\overline{\delta}_n) \le \eta R_A(x,\overline{\delta}_n)$ for some $\eta < 1$, the difference $R_A(x,\overline{\delta}_n) - R_B(x,\overline{\delta}_n)$ is bounded below by a fixed positive fraction of $\mathrm{MB}_n(x,\overline{\delta}_n)$. Therefore the last display is strictly negative for all sufficiently large n, contradicting $F_n(x,\overline{\delta}_n) = 0$. Thus $x > 2\kappa/n$.

At any point on the upper δ-boundary with $F_n(x,\overline{\delta}_n) = 0$, we therefore have

$$
\mathrm{MB}_n(x,\overline{\delta}_n) < C'(\kappa/n) < C'(x),
$$

where the second inequality follows from $x > 2\kappa/n$ and the strict monotonicity of C'. Hence

$$
G_n(x,\overline{\delta}_n) < 0.
$$

Step 3: Construction of an inefficient equilibrium sequence. We construct an equilibrium sequence in which A is elected with probability converging to one in both states.

Applying the generalized Poincaré–Miranda theorem (Lemma 2) to F_n and G_n on the domain D_n yields, for every sufficiently large n, an interior point

$$
(x_n, \delta_n) \in D_n
$$

such that

$$
F_n(x_n,\delta_n) = G_n(x_n,\delta_n) = 0.
$$

By the discussion above, these two equations are exactly the two equilibrium conditions in Lemma 1. Hence $\alpha(x_n,\delta_n)$ is a symmetric equilibrium with information acquisition.

Since the root lies in the interior of D_n, we have

$$
0 < x_n < Mn^{-1/2} \qquad \text{and} \qquad \delta_n > 3Mn^{-1/2}.
$$

Thus $\delta_n > 2x_n$ for all sufficiently large n. Using (4),

$$\pi(z_B; \alpha(x_n, \delta_n)) = \frac{1}{2} + \frac{\delta_n}{2} - (1 - \delta_n)x_n > \frac{1}{2}.$$

This implies that the probability of A being elected in state z_B is strictly larger than $\frac{1}{2}$ for all sufficiently large n. The sequence of outcome-probability pairs

$$(\Pr(A \text{ is elected} \mid z_A; \alpha(x_n, \delta_n)), \Pr(A \text{ is elected} \mid z_B; \alpha(x_n, \delta_n)))$$

has a convergent subsequence by compactness. Along any such subsequence, the probability that B is elected in z_B cannot converge to one. Therefore the subsequence cannot have asymptotically efficient outcomes. By the third part of Theorem 1, along this convergent subsequence, A must be elected with probability converging to one in both states. Relabeling this subsequence gives the desired equilibrium sequence with asymptotically inefficient outcomes.

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