Information Frictions and Opposed Political Interests

Carl Heese, Vienna

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It is costly to filter and pay attention to all relevant information in most collective choice settings ("information frictions")

Voters invest time and effort to cast an informed vote.
Voter information websites, TV debates of candidates, ...

Competition between opposed interests and political outcomes may be shaped by such informational efforts and frictions

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- "Flavour": Interest groups are not "organised" or fully coordinated
- Applies to groups with coordination constraints, such as large groups of voters in elections.

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- "Flavour": Interest groups are not "organised" or fully coordinated
  compare, e.g., with the common agency models (Grossman & Helpman, '97)

- Applies to groups with coordination constraints, such as large groups of voters in elections.
  Similar in spirit: Participation games (Palfrey and Rosenthal, 1983)

Model

drawing from Feddersen&Pesendorfer (1997); Bhattacharya (2013), Martinelli (2006)

Simple majority election

$2n + 1$ voters

Policies A and B

State of the world α and β

Private i.i.d. types capture prior, info cost, and state-dependent utilities

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$t = (p, r, u(z, \omega)_{z \in \{A,B\}, \omega \in \{\alpha,\beta\}}) \in T = [0, 1] \times \mathbb{R}_{>0} \times \mathbb{R}^4 \sim H$

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Types $t = (p, k)$ drawn i.i.d. from distribution H with support on a compact rectangle and positive, continuous density on the support.

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Information Frictions: voters can acquire private signal about state with $\Pr(a|\alpha) = \Pr(b|\beta) = \frac{1}{2} + x$ at cost $c(x) = \frac{x^d}{d}$.

Two interest groups with opposed interests

Interest Group: types with same preference ranking of policies given the state.

The types with $k > 0$ are a majority in expectation (mass $\lambda > \frac{1}{2}$)

The types with $k < 0$ are a minority in expectation (mass $1 - \lambda < \frac{1}{2}$)

Summary of the Voting Game and Strategies

1. State α or β
2. Each voter privately observes type t
3. Voters simultaneously choose signal precision according to

$$x : T \to [0, \frac{1}{2}]$$

4. Voters observe their signal realizations, then simultaneously vote A or B according to

$$\mu : T \times \{a, b\} \to \{A, B\}$$

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Study Bayes-Nash equilibrium $\sigma = (x, \mu)$ in symmetric, pure strategies, and with information acquisition, i.e. $x \neq 0$.

Applications and Intepretations of the Model

1. There are **natural applications**:

   - Two candidates, by and large centrists, stand for election and voters are uncertain if A is more left (state α) or B is more left (state β). Leftists prefer A only in α, rightists only in β.
     (see Meirowitz, 2006; Bhattacharya, 2013)

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2. Conjecture: Model is the **reduced form** of a model with **arbitrarily many finite states**, making it amenable to more applications.

Result: Effort Ratio

$\mathcal{H}$ is the domain of admissible type distributions.

Focus on equilibrium sequence $(\sigma_n)_{n\in\mathbb{N}}$ satisfying a condition necessary for the asymptotic outcome distribution to vary with the state (write: *informative*).[1]

[1] The condition is equivalent to $\lim_{n\to\infty}\left[\int_t x_n(t)dH(t)\right]/n^{-\frac{1}{2}} \in \mathbb{R}$.

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Lemma. There is $R : \mathcal{H} \to \mathbb{R}$ so that for all *informative* equilibrium sequences $(\sigma_n^*)_{n\in\mathbb{N}}$, the limit ratio of the aggregate informational effort is given by

$$R(H) = \lim_{n\to\infty} \frac{\int_{t=(p,k):k>0} x(t)dH(t)}{\int_{t=(p,k):k<0} x(t)dH(t)}$$

if $R \neq 1$.

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Main Result: Informational Contest

Theorem. There is an equilibrium sequence $(\sigma_n^*)_{n\in\mathbb{N}}$ in which the policy preferred by the interest group with the higher aggregate informational effort is elected,

$$\lim_{n\to\infty} \Pr(A \text{ wins}|\alpha;\sigma_n^*) = \lim_{n\to\infty} \Pr(B \text{ wins}|\beta;\sigma_n^*) = 1 \text{ if } R(H) > 1.$$
$$\lim_{n\to\infty} \Pr(A \text{ wins}|\alpha;\sigma_n^*) = \lim_{n\to\infty} \Pr(B \text{ wins}|\beta;\sigma_n^*) = 0 \text{ if } R(H) < 1.$$

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- No full-information equivalence: Minority group wins if intensities large. *Similar to the efficiency results in costly voting and vote-buying (Krishna and Morgan, 2011; Eguia and Xefteris, 2021))*
- Taking R as a measure of "influence", will see: influence depends on group size, intensities, priors, and information cost in non-trivial ways.

Example

An example illustrates central ideas.

Instrumental Voting and Pivotality

A voter is **pivotal** if n others are voting A and n others are voting B

Given any σ, **best response** with signal s and type $t = (p, k)$ with $k > 0$ is to vote A iff

$$\Pr(\alpha|\mathrm{piv}, s; t) - \Pr(\beta|\mathrm{piv}, s; t) \geq 0$$

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or, equivalently, if

$$\frac{p}{1-p} \geq \frac{\Pr(s|\beta)}{\Pr(s|\alpha)} \frac{\Pr(\mathrm{piv}|\beta)}{\Pr(\mathrm{piv}|\alpha)}.$$

Example: Distribution with 6 types

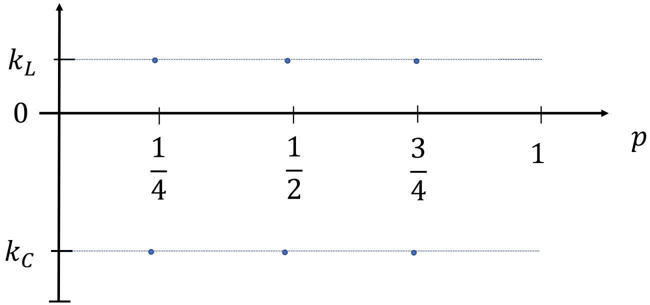

Figure shows 6 types $t = (p, k)$ (as dots)

Upper 3 types prefer A in α and B in β ($k > 0$)

Lower 3 types prefer B in α and A in β ($k < 0$)

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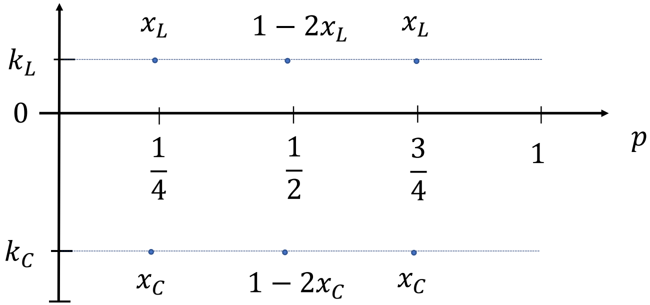

Upper row shows **conditional likelihood** of types with $k > 0$

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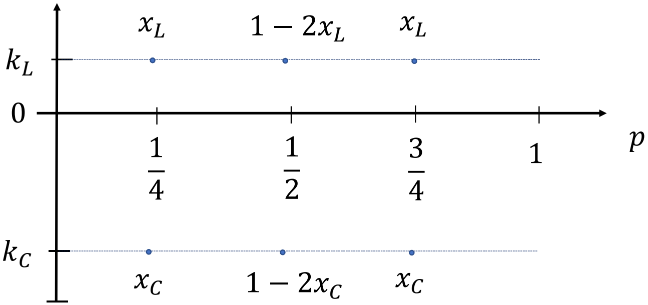

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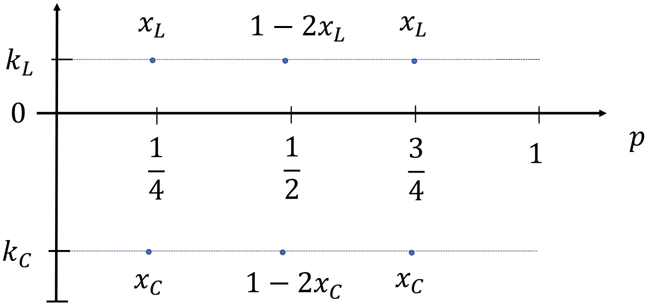
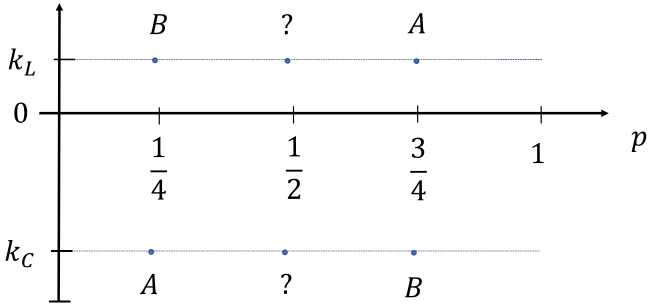


Upper row shows **conditional likelihood** of types with $k > 0$

Lower row shows conditional likelihood of types with $k < 0$

The types with prior $1/4$ and $3/4$ are **equally likely**

Optimal Choices Without Costly Information



Symmetry of types implies unique equilibrium, which satisfies $\frac{\Pr(\text{piv}|\beta)}{\Pr(\text{piv}|\alpha)} = 1$.

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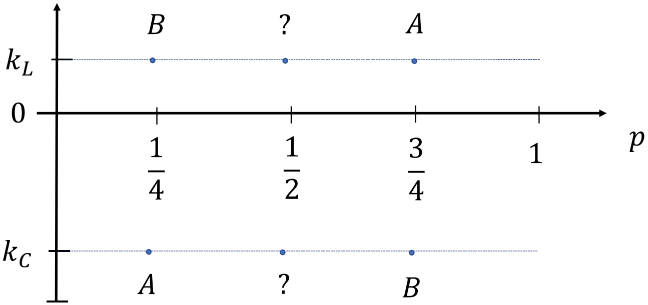

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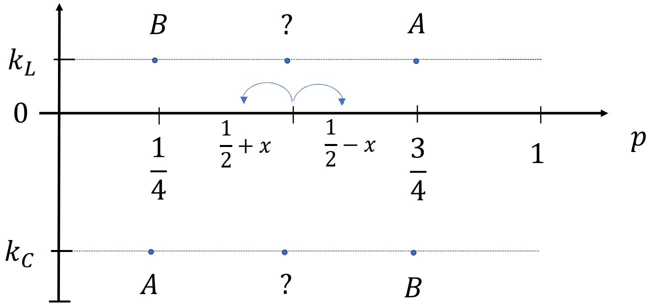
The types with prior $1/4$ and $3/4$ have a strict preference without signal.

Types with median prior are **indifferent** without signal.

Only Types with Median Prior Acquire Information



Value of information goes to zero as the electorate grows large.

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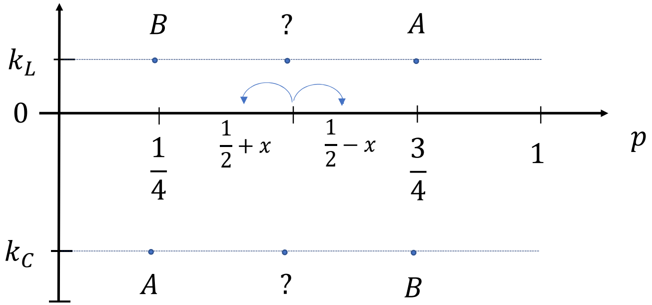

Value of information goes to zero as the electorate grows large.

Only signals with precision x larger $1/4$ flip the strict preference of the types with prior $1/4$ and $3/4$.

Cost for these signals exceed benefits.

An Informational Contest

Two phenomena (which will also be central in general)

"Miscoordination of the uninformed":
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Difference in vote shares between A and B driven by choices of x. In fact,

$$\int_{t=(p,k):k>0} x(t)dH(t) - \int_{t=(p,k):k<0} x(t)dH(t)$$

The effort ratio R

The median types choice of x equates marginal benefits with costs

$$\Pr(\mathrm{piv}|\sigma)k = C'(x) \Leftrightarrow x = \left[\Pr(\mathrm{piv}|\sigma)k\right]^{\frac{1}{d-1}}$$

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So,

$$R(H) = \frac{\mathsf{Pr}_H(t = (1/2, k_L))}{\mathsf{Pr}_H(t = (1/2, k_C))}\left[\frac{k_L}{k_C}\right]^{\frac{1}{d-1}}$$

The **likelihood of the median types** and the **intensities** matter.

The elasticity d of the cost function modulates how both factors **substitute**.

Numerical Calculations: Finite Electorate

Fix $2\boldsymbol{n}+1=31$, $\lambda=\frac{1}{3}$, $k_L=1$, and $x_C=x_L=0$.

The table shows the likelihood of outcome A in α and B in β in the (unique) equilibrium for **varying cost elasticities** d **and intensities** k_C.

k_C	$d=2$	$d=3$
0	0.79	0.94
1	0.65	0.86
2	0.5	0.77
3	0.35	0.75
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The minority dominates the election with high intensities $k_C=4$ when $d=2$, but not when $d=3$.

Intensities matter more when information is more "expensive"—as measured by higher d.

Continuous Type Distributions

Continuous distribution of types (p, k)

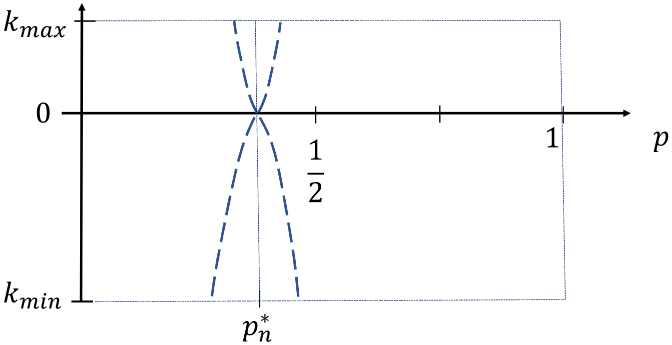

Figure illustrates a typical equilibrium.

There is p_n^* so that types with prior p_n^* are indifferent without signal.

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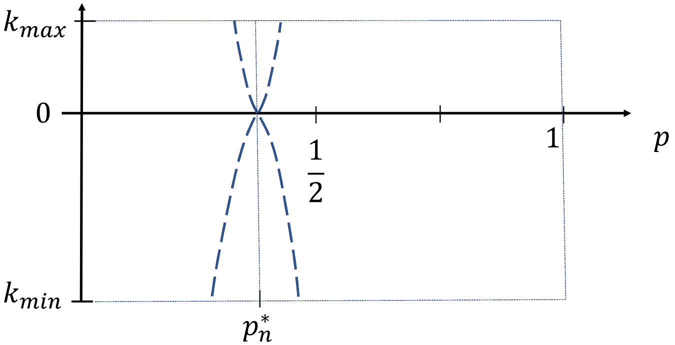

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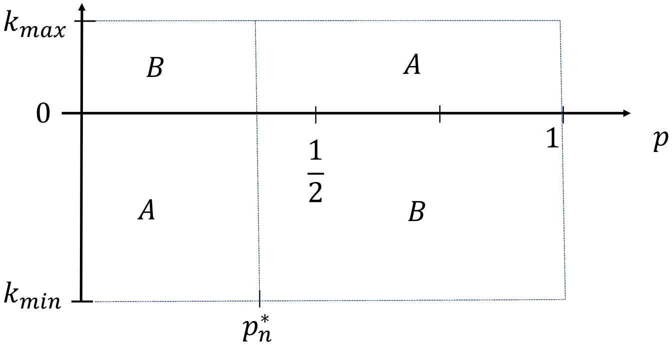
Types in **cone** around indifferent types acquire information, $x(t) > 0$.

Optimal Choices Without Costly Signal



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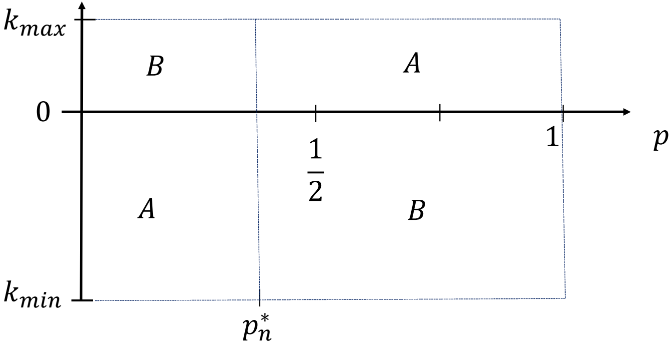

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Example: Uniform type distribution in the figure.

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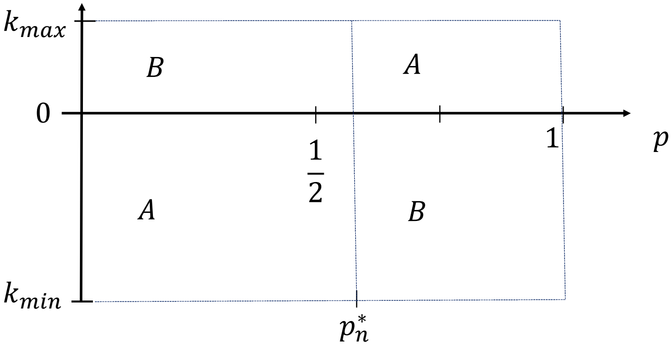

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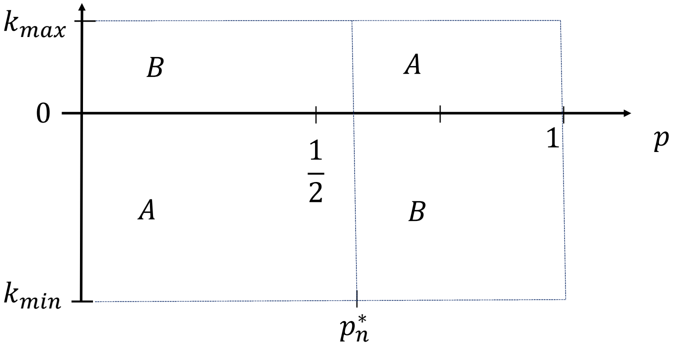
Example: Uniform type distribution in the figure.

Optimal Choices For Varying Beliefs About the State



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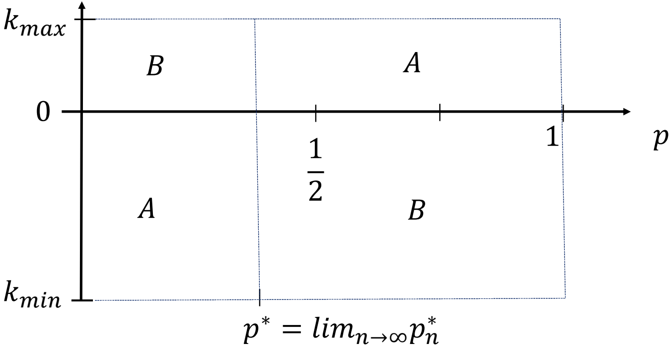
[2] It is well-known that monotonicity violations complicate the strategic setting; e.g. they imply equilibrium multiplicities in the standard setting in Bhattacharya, 2013.

Miscoordination of the Uninformed and Close Elections



p^* is pinned down uniquely: this is implied by monotone preferences and,

Lemma. For any *informative* equilibrium sequence, the expected vote shares converge to 50% for both A and B, as $n \to \infty$, in each state.

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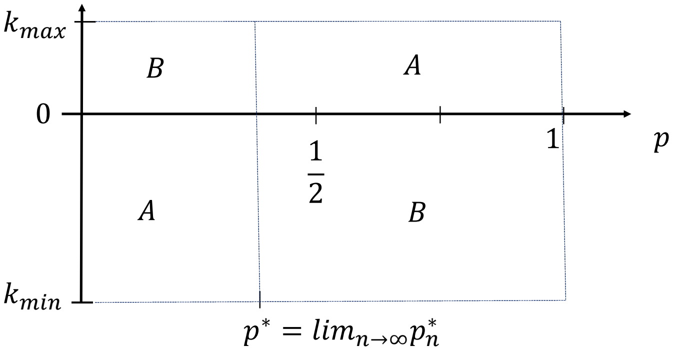

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Intuition: the election has to be close to being tied so that voters have sufficient incentives to acquire information. (link to formal argument)

The effort ratio R

Lemma. The limit ratio of the aggregate information effort is

$$R(H) = \underbrace{\frac{\Pr(\{t=(p,k)|k>0, p=p^*\}}{\Pr(\{t=(p,k)|k<0, p=p^*\}}}_{\text{Likelihood of limit indifferent type}} \cdot \underbrace{\frac{\mathrm{E}(k^\kappa|k>0, p=p^*)}{\mathrm{E}(k^\kappa|k<0, p=p^*)}}_{\kappa\text{-measured intensity}}$$

for $\kappa = \frac{2}{d-1}$, where $C(X) = \frac{X^d}{d}$.

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2. Special interests, i.e., if much welfare is at stake.
2. Few conflicts of opinion; formally, if the dispersion of priors is low.

The effort ratio R

Lemma. The limit ratio of the aggregate information effort is

$$R(H) \quad = \quad \underbrace{\frac{\Pr(\{t=(p,k)|k>0, p=p^*\}}{\Pr(\{t=(p,k)|k<0, p=p^*\}}}_{\text{Likelihood of limit indifferent type}} \cdot \underbrace{\frac{\mathrm{E}(k^\kappa|k>0, p=p^*)}{\mathrm{E}(k^\kappa|k<0, p=p^*)}}_{\kappa\text{-measured intensity}}$$

for $\kappa = \frac{2}{d-1}$, where $C(X) = \frac{X^d}{d}$.

Limit result is "too extreme". View it as "giving directions". A group's influence is large when the group has

1. Information advantages, e.g., due to insider access.
2. Special interests, i.e., if much welfare is at stake.
2. Few conflicts of opinion; formally, if the dispersion of priors is low.

There are non-trivial interactions, e.g. between the cost function and the intensities (recall the example).

Other Results

All Equilibria

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- In the medium effort equilibrium, limit outcomes ($n \to \infty$) depend on both the effort ratio R and the preferences given the prior beliefs.
- In the low effort equilibrium, limit outcomes are pinned down by the preferences given the prior beliefs.

Equilibrium Existence: a Fixed Point Theorem

Prove and utilize a generalization of the **Poincaré-Miranda theorem**, a fixed point theorem equivalent to Brouwer's. (Ekmekci, Heese et. al, 2021)

Our version relaxes the conditions under which it applies and this is necessary for the purposes of this paper.

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Common value setting (i.e. $\lambda = 1$)

Common value settings have been analyzed extensively, mostly with a focus on information aggregation
Feddersen & Pesendorer, '98; Martinelli, 2006, etc.

Observation: **Competition upsets information aggregation**; minorities may win.

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Companion paper with pure common values setting: Show that

- the equilibrium **multiplicity is not caused by competition** but due to strategic complementarities in information choices,
- Information aggregation may fail due to mis-coordination on an inefficient equilibrium even if all voters share a common type.

Conclusion

- Model of competition between two interest groups via info efforts.

- The effort ratio is constant across (informative) equilibria when the groups are large.
- The absolute effort varies across equilibria.
- The effort-maximal equilibrium is like an "informational contest": the group with the larger effort "wins".
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  - information advantages,
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- Taking the endogenous effort ratio as a measure of influence, characterizing the effort ratio, we have shown how
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  and interaction effects matter.
- We discussed the relation to the information aggregation literature.

Expected Vote Shares and Pivot Probabilities

The expected vote share for A in state $\omega \in \{\alpha, \beta\}$ is

$$q(\omega; \sigma_n) = \int_t (\frac{1}{2} + x(t))\mu(t, a) + (\frac{1}{2} - x(t))\mu(t, b) dH(t).$$

Given a strategy σ, the likelihood of the pivotal event in state ω is

$$\Pr(\mathrm{piv}|\omega; \sigma, n) \approx \binom{2n}{n} \Big[q(\omega; \sigma)(1 - q_q(\omega; \sigma)) \Big]^n$$

Informative Equilibrium Sequences

For any sequence of strategies $(\sigma_n)_{n\in\mathbb{N}}$ and any n, let

$$\delta_n(\omega;\sigma_n) = \frac{q(\omega;\sigma_n) - \frac{n}{2n+1}}{s(\omega;\sigma_n)}. \tag{1}$$

This measures the distance between the expected vote share and the majority threshold in multiples of the standard deviation $s(\omega;\sigma_n)$ of the vote share distribution for $\omega \in \{\alpha, \beta\}$, where $s(\omega;\sigma_n)^{-1} = \sqrt{\frac{(2n+1)}{q(\omega;\sigma_n)(1-q(\omega;\sigma_n))}}$.

An equilibrium sequence is *informative* if

$$\lim_{n\to\infty} \delta_n(\alpha;\sigma_n) - \delta_n(\beta;\sigma_n) \neq 0. \tag{2}$$

Asymptotic distribution of the outcome

A normal approximation of the distribution of the number of A-votes

$$\lim_{n\to\infty} \Pr(A|\omega;\sigma_n) = \lim_{n\to\infty} 1 - \Phi(-\delta_n(\omega;\sigma_n)). \quad (3)$$

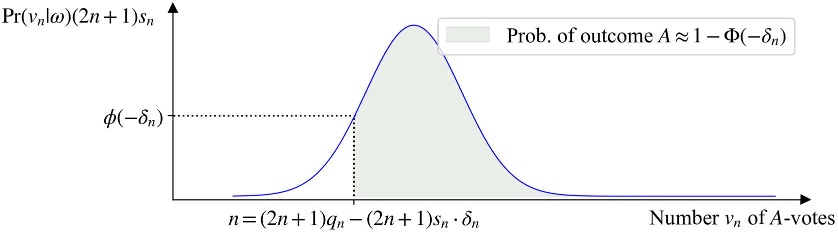

Figure: Normal Approxmation: The Binomial has mean $(2n+1)q_n$ for $q_n = q(\omega;\sigma_n)$ and standard deviation $(2n+1)s_n = ((2n+1)(q_n(1-q_n))^{\frac{1}{2}}$ for $s_n = s(\omega;\sigma_n)$. The outcome is A if there are more than n votes for A.

Monotone Preferences

Before observing the private signal s, a type $t = (p, k)$ is indifferent between voting A and B if

$$p = \frac{\Pr(\mathrm{piv}|\beta; \sigma, n)}{\Pr(\mathrm{piv}; \sigma, n)}.$$

The voters have *monotone preferences* if the distribution $H \in \mathcal{H}$ is so that the function

$$\begin{aligned}\Psi(q) \quad &= \quad \Pr(\{t = (p, k) : k > 0\}) \Pr(\{t : p > q | k > 0) \\ &\quad - \Pr(\{t = (p, k) : k < 0\}) \Pr(\{t : p < q | k < 0\})\end{aligned}$$

is strictly increasing in q.

Stirling Approximation of the Pivotal Likelihood

back

A Stirling approximation of the pivotal likelihood gives

$$\Pr(\mathrm{piv}|\omega;\sigma,n) \approx 4^n(n\pi)^{-\frac{1}{2}}\left[q_n(1-q_n))\right]^n,$$

for $q_n = q(\omega;\sigma_n)$. Thus, unless $q_n(1-q_n) \to \frac{1}{4}$, the term is exponentially small.